On Different Measures of Labor Market Slack: Through the Lens of the Phillips Curve

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Abstract

During the COVID period, the vacancy-to-unemployment ratio (V/U) emerged as a more accurate measure of labor market tightness than the unemployment rate (U). However, some argue that this claim may be over-fitting the COVID-19 pandemic episode. This paper addresses this critique by using pre-pandemic time-series data and exploits better identification from Metropolitan Statistical Area (MSA)-level panel data. I construct state-space models and apply the Kalman filter to MSA-level panel data, thereby jointly estimating a non-linear Phillips curve and time-varying natural rates of U and V/U. Time series data alone cannot distinguish between the two measures on pre-pandemic data, but panel data indicates that V/U is superior. These findings show that V/U was a better measure of economic slack even before the pandemic, and suggest a greater role for it in economic forecasting and monetary policy.

JEL Classification: E31, J6

Keywords: Inflation, Slack Measures, Phillips curve, Beveridge curve

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1 Introduction

The Phillips curve (PC) is a central concept in macroeconomics that links inflation to labor market slack. Accurately measuring labor market conditions is a key challenge in estimating the PC. Historically, since Phillips (1958), there has been a tradition of using the unemployment rate or its deviation from the natural rate as the measure of slack, a practice that continues throughout much of the modern literature.

During the COVID-19 pandemic in 2021, a significant puzzle emerged. Inflation rose sharply, even though the unemployment rate did not change much. This anomaly implies a possible breakdown in the traditional Phillips curve relationship. An idea proposed by some economists, which quickly gained popularity, was to replace the unemployment rate with the vacancy-to-unemployment ratio (V/U) as the measure of labor market tightness.

This measure has a theoretical appeal because it is the variable that affects wage pressures in search and matching models. Empirically, during the pandemic, vacancies increased significantly, so V/U rose sharply even though U did not change much. By this measure, the labor market was tight, which would explain why inflation rose. Then, V/U quickly became a favored variable in many recent papers (Ball, Leigh and Mishra, 2022; Benigno and Eggertsson, 2023; Blanchard and Bernanke, 2023; Cecchetti et al., 2023; Barnichon and Shapiro, 2024).

However, this shift in the measure of labor market slack has faced criticism. Much of the evidence for the superiority of V/U comes from the COVID period, raising doubts about its effectiveness across broader historical contexts (Şahin, 2022). The focus on V/U may be too specific to post-2021 economic conditions, suggesting that it fits this unique period rather than being a universally superior measure. Hence, changing the Phillips curve specification based on just this three-year period risks being seen as "cherry-picking" evidence.

This paper aims to provide new evidence on the most accurate measure of labor market slack, to determine whether V/U is generally the better measure or merely a result of over-fitting during the COVID period. To achieve this, I extend the empirical work in two directions.

First, I allow for time-varying natural rates for the V/U ratio, similar to how time-varying natural rates of unemployment are considered. Traditionally, the non-accelerating inflation rate of unemployment (NAIRU) was seen as constant until studies in the 1990s began to recognize its variability over time. Acknowledging that the natural rate of unemployment changes over time suggests that the natural rate of V/U might also vary.

Following Staiger, Stock and Watson's (1997*a*) work on time-varying natural unemployment rates, I investigate a time-varying natural rate for V/U, ensuring both measures are evaluated on comparable grounds.

Implementing this time variation in natural rates presents a challenge. Recent literature finds that the Phillips curve appears to be non-linear, as inflation rises very sharply at high levels of V/U (Ball, Leigh and Mishra, 2022). I contribute to the literature by using state-space models to estimate the natural rates and the Phillips curve coefficients simultaneously, extending the time-varying natural rate concept to non-linear models.

The second way I advance the empirical work is by using regional-level data, a strategy gaining popularity due to endogeneity problems highlighted in Mavroeidis, Plagborg-Møller and Stock (2014). They demonstrate that aggregate macroeconomic data struggle with identification challenges, obscuring empirical insights on the Phillips curve due to limited information. As a result, studies such as Babb and Detmeister (2017), Fitzgerald et al. (Forthcoming), Hazell et al. (2022), Hooper, Mishkin and Sufi (2020), and McLeay and Tenreyro (2020) have moved toward using regional data with two-way fixed effects.

In contrast to aggregate data studies, previous research has not used state-space models to estimate natural rates for slack measures in a panel setup. In this study, I allow each metropolitan statistical area to have its own natural rate of slack measures, where the natural rates can evolve differently across regions.

In my research, I find that in aggregate-level analyses, V/U only outperforms the unemployment rate when pandemic period observations are included; it does not outperform in earlier periods. Therefore, Şahin's (2022) critique about "cherry-picking" evidence is valid. In contrast, the regional analysis provides stronger support for V/U, even when excluding the pandemic period, indicating that the effectiveness of V/U is not merely specific to the COVID period data.

Finally, motivated by these empirical results, I introduce a short-run theoretical model to demonstrates why the relationship between U and inflation is unstable, and why V/U naturally appears in the relationship. I build on the Diamond-Mortensen-Pissarides (DMP) model, where V/U is a critical indicator of labor market tightness. I incorporate this ratio into macro-econometric models to illustrate its impact on inflation. Drawing from the works of Blanchard and Katz (1996) and Blanchard and Bernanke (2023), I develop an expectations-augmented, reduced-form Phillips curve.

Layout. The paper is organized as follows. Section 2 explains the inflation anomaly and the proposed resolution using V/U. Section 3 presents aggregate data analysis. Section 4 discusses identification challenges and offers regional empirical evidence supporting

V/U. Section 5 examines the wage Phillips curve. Section 6 details the theoretical model incorporating the V/U ratio into the Phillips curve. Section 7 concludes. Additional empirical findings are included in the appendix.

1.1 Literature

My paper adds to and builds on the rich literature on the Phillips curve.

Phillips Curve and Natural Rate. This work directly builds on the traditional Phillips curve literature. Since the introduction by Phillips (1958), the unemployment rate has been widely used, partly due to its simplicity. Yet, the natural rate was seen as constant for a long time until it was being recognized as time-varying. Previous research, including works by Barnichon and Matthes (2017); Brauer (2007); Crump et al. (2019); Fabiani and Mestre (2004); Gordon (1997); Staiger, Stock and Watson (1997*a*), typically focused on aggregate data to estimate time-varying natural unemployment rates. In contrast, this study also estimates natural rates with nonlinear Phillips curve, and adopts a regional panel data framework.

Alternative Measures. Another branch of literature explores alternatives to the unemployment rate for measuring labor market slack. Early on, Medoff and Abraham (1981) suggested the relevance of vacancies, yet the idea initially received limited attention. Recent studies, including Faberman et al. (2020); Hall and Schulhofer-Wohl (2018); Hornstein, Kudlyak and Lange (2014), have sought to refine slack measures. Abraham, Haltiwanger and Rendell (2020) proposed a sophisticated measure of labor market tightness, adjusting vacancies for the effective search effort across different groups.

The pandemic intensified efforts to find alternative labor market slack measures. Interest grew in job openings and quits rates, but the V/U ratio gained prominence due to its ties to search and matching theory. Recent work like Furman and Powell (2021) assessed different labor market measures within Phillips curve models, and identified the quits rate as crucial for explaining wage growth, and found the unemployed-to-job openings ratio to be a key predictor for core CPI.

In addition, Barnichon and Shapiro (2022) explored the forecasting power of different measures, noting the V/U ratio's superiority over the unemployment rate. Barnichon and Shapiro (2024) further demonstrated that shifts in the Beveridge curve can contribute to inflation. Domash and Summers (2022*a*,*b*) found that both unemployment rate and vacancies are key predictors of wage inflation and developed a firm-side predicted unemployment rate using these metrics. They contend that this firm-side rate matches the explanatory power of the actual unemployment rate for wage inflation dynamics.

Cross-sectional Phillips Curve. This growing empirical branch within Phillips curve literature highlights identification challenges with aggregate data and suggests cross-sectional data as a solution to reduce bias in national estimates.

Hazell et al. (2022) review recent cross-sectional studies, underscoring national-level challenges such as monetary policy endogeneity, difficulty in measuring inflation expectations and their correlation with slack, and distinguishing between demand and supply shocks. They constructed new state-level inflation measures (headline, non-tradable, and tradable) and estimated the Phillips curve at the state level. Their findings suggest that changes in inflation dynamics are significantly influenced by the long-term inflation expectations, documenting the flattening of the Phillips curve.

Several researchers underscore the value of leveraging regional variations for better identification. McLeay and Tenreyro (2020) and Fitzgerald et al. (Forthcoming) demonstrate that monetary policy biases national time-series studies on the Phillips curve relationship. They suggest that cross-sectional data can unveil a stronger link between unemployment or output gaps and inflation, given how central bank optimal actions can obscure the national-level Phillips curve. Similarly, Hooper, Mishkin and Sufi (2020) finds that regional inflation data can mitigate aggregate time-series biases.

Moreover, studies like Babb and Detmeister (2017) and Kiley (2014) use metropolitan or city-level data for Phillips curve estimations. Another segment focuses on the producer price index (Firat, 2022*a*; Heise, Karahan and Şahin, 2022). This literature collectively highlights the importance of gathering stronger evidence from disaggregated data.

Theoretical Work. This paper's theoretical model draws from the Diamond-Mortensen-Pissarides (DMP) framework. Seminal works by Pissarides (1985) and Mortensen and Pissarides (1994) emphasize the V/U ratio's critical role in the labor market's search and matching process. This ratio, a key indicator of labor market tightness, directly impacts job-finding and job-filling rates, as well as wage negotiations. My model, while rooted in the DMP framework's concepts of endogenous job dynamics, specifically addresses short-run business cycle variations with exogenously set number of jobs.

Further, the model integrates the aggregate wage-price determination approach from Blanchard and Bernanke (2023) and Blanchard and Katz (1996), focusing on connecting Nash-bargain wage to price inflation in the simplest way. This approach differs from Blanchard and Galí (2010)'s dynamic stochastic general equilibrium model that incorporates search frictions within a New Keynesian context.

2 The Inflation Anomaly

The Phillips curve has long served as the stylized model for analyzing inflation, depicted as:

$$\pi_t = \pi_t^e - f\left(\text{slack}_t\right) + \nu_t, \tag{1}$$

where inflation is determined by expectations, labor market slack, and supply shocks (ν_t).

The Anomaly Historically, the unemployment rate was the standard and go-to metric for labor market slack. The sudden inflation increase in 2021, however, caught many by surprise, challenging the prevailing theory.

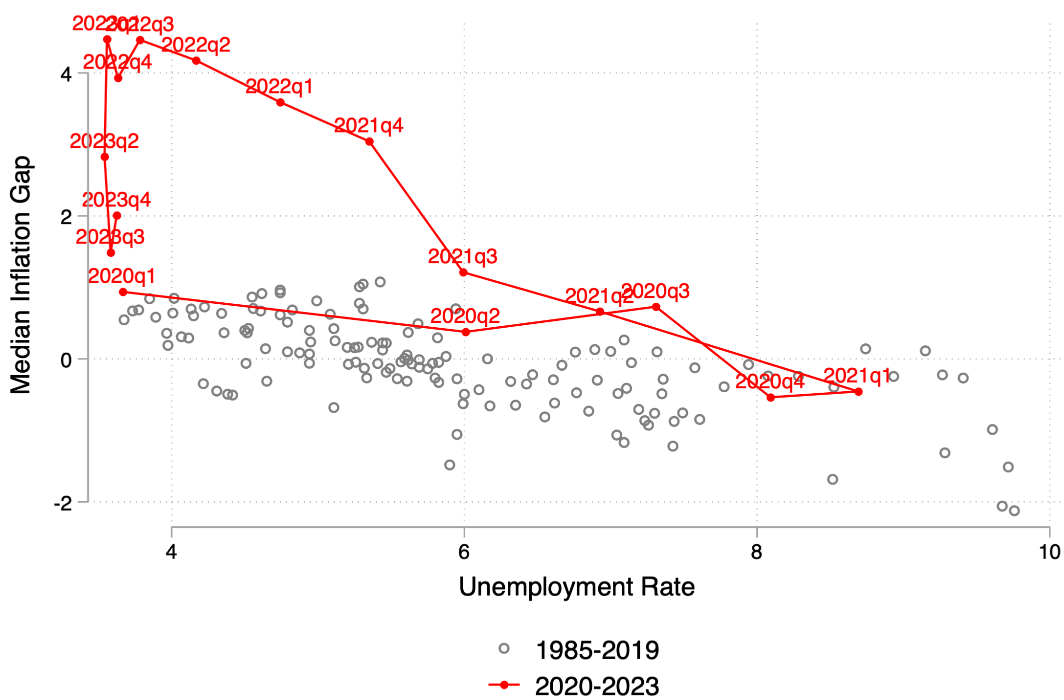


Figure 1: Phillips Curves based on Unemployment Rate

Figure 1 displays a scatter plot of the inflation gap, defined as median CPI inflation minus the SPF 10-year forecast, against the unemployment rate. I investigate the inflation gap's trajectory against the unemployment rate since 1985. Using quarterly data, I compare the inflation gap with the unemployment rate's 4-quarter average (current and the preceding three quarters), effectively considering the lagged impact of labor market slack on inflation. Different markers represent various sample periods; the first spans from 1985 to 2019, pre-pandemic, and the second from 2020 to 2023, the COVID era and beyond. The first sample comprises of observations from 1985 to 2019, noted for a long period of low inflation and a soft labor market on average. The subsequent period, 2020 to 2022, known as the COVID era, experienced heightened inflation and tightened labor market.

From 1985, the data show a downward-sloping linear relationship between the inflation gap and unemployment rate, consistent with the standard expectations-augmented Phillips curve. However, a significant shift occurred in 2021, with newer data points clustering in the graph's upper-left, marked by not particularly low unemployment rates but elevated values of inflation gap. These conditions, high inflation alongside moderate unemployment rates, were unparalleled in the post-1985 national time-series, baffling numerous economists and policymakers. This inflation anomaly has thus reignited the discussion on the most accurate labor market slack indicator.

The Proposed Resolution. The quest to reconcile high inflation with traditional Phillips curve relationships has gained momentum.

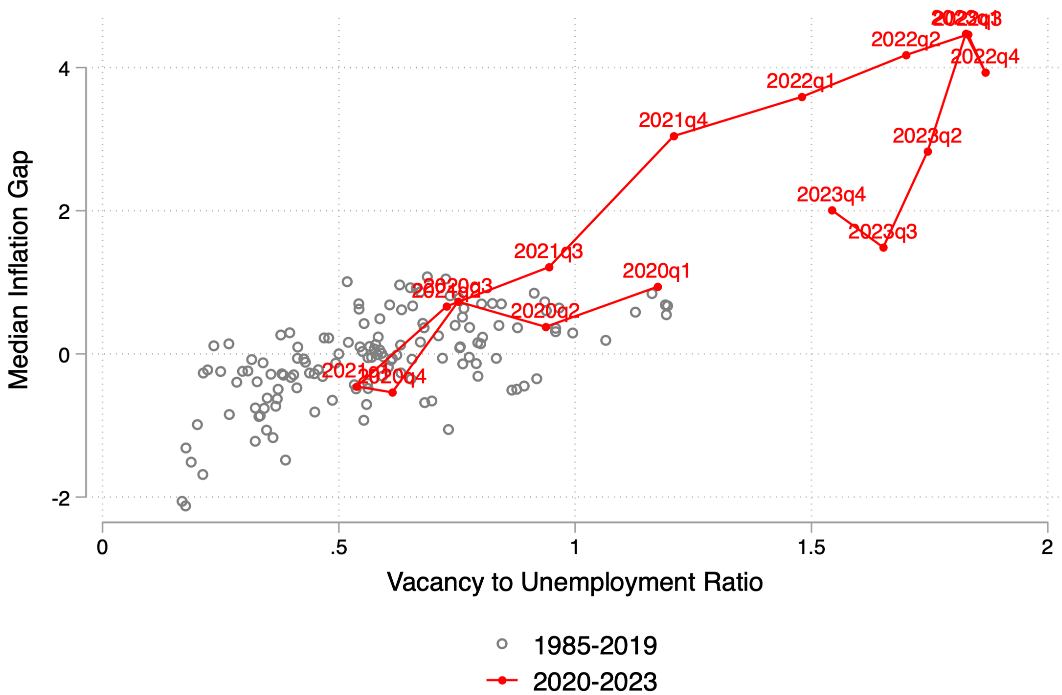


Figure 2: Phillips Curves based on V/U Ratio

Since the fourth quarter of 2000, the Bureau of Labor Statistics (BLS) introduced the Job

Openings and Turnover Survey (JOLTS), detailing labor market conditions, which was initially overlooked but has recently garnered attention. Among alternative measures, the V/U ratio gradually gains its popularity, emerging as the most effective, explaining the unusual inflation pattern over three years and directly linking to the labor market's search and matching model.

To clarify the empirical superiority of the V/U ratio, Figure 2 presents a scatter plot of the inflation gap against the V/U ratio. Similar to the previous figure, it uses different markers for the periods 1985–2019 and 2020–2023. The pre-pandemic era is represented in the lower-left corner, and the COVID era in the upper-right. Data from the pandemic period appear in the upper-right corner, where inflation rates are considerably higher than in the pre-pandemic period, corresponding with higher values of V/U. The V/U ratios are also significantly higher than in the pre-pandemic periods, suggesting much greater labor market tightness.

Unlike the figure for the unemployment rate, the V/U data reveal a consistent link between the inflation gap and labor market tightness, with less noise in the relationship. Lower V/U values are associated with smaller inflation gaps, and higher values with larger gaps. The Phillips curve model using the V/U ratio effectively reduces the anomaly, maintaining a stable relationship before and after COVID. This consistency has made the V/U ratio a preferred measure in many recent studies.

3 V/U vs. U: Aggregate Analysis

The next two sections aim to assess the explanatory and identification power of the V/U ratio versus the unemployment rate for inflation within a price Phillips curve framework. Initially, the analysis explores fixed natural rate contexts before transitioning to time-varying natural rates. Then, the subsequent section examines their effectiveness in wage Phillips curve scenarios. This investigation is conducted at both the national and regional levels.

Within the Phillips curve framework, as shown in equation (1), comparing two slack measures involves three critical considerations when formulating the regression model: selecting an appropriate core inflation measure, defining the sample period to exclude the effects of the pandemic years, and recognizing the importance of non-linearity. Before presenting the primary regression model for the aggregate-level analysis, I will discuss these aspects.

Measuring Inflation and Inflation Expectations. The conventional core inflation measure excludes food and energy from the headline inflation. However, significant price shocks are not confined to these sectors alone. The median CPI, an approach that excludes outliers across all industries, emerges as a more effective alternative for filtering out significant price changes, as emphasized by studies like Bryan, Cecchetti and Wiggins (1997) and Ball and Mazumder (2019*a*,*b*). This paper thus uses the median CPI as the core inflation measure. [1]

Recent research, including Hazell et al. (2022), re-emphasizes the long-recognized importance of inflation expectations in determining inflation. This study uses the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia for survey-based inflation expectations. The SPF provides 10-year-ahead inflation expectations, which are less correlated with current economic conditions and cover a long time period as Ball, Leigh and Mishra (2022) extended this series back to the 1980s.

Data and Estimation Period. To separate the impact of pandemic-related observations while also utilizing an extended historical dataset for national level time series analysis, this study defines the sample period from 1985 to 2019 as the pre-pandemic era. It then expands the analysis to include data up to 2023, thereby encompassing the effects of recent labor market events.

[1] Ball, Leigh and Mishra (2022) finds that the Phillips curve fits much better with median inflation on the left-hand side than with conventional core inflation.

The Bureau of Labor Statistics (BLS) began publishing Job Openings and Labor Turnover Survey (JOLTS) data in December 2000. In line with existing research, vacancy rates have been extended back to 1985 using the help-wanted index from Barnichon (2010), an approach this paper adopts. Barnichon (2010) provides a longer time series for vacancy rates dating back to 1951, using help-wanted advertisements to approximate actual V/U ratios effectively. Utilizing this extended vacancy data, this study compiles a dataset spanning from 1985 to 2019 for the pre-pandemic period and a broader dataset from 1985 to 2023 on a quarterly basis. [2]

Non-linearity. The non-linear impact of labor market slack on inflation has been highlighted in numerous studies, including the foundational work of Phillips (1958). Hooper, Mishkin and Sufi (2020) find empirical evidence that the Phillips curve becomes steeper when the labor market is tighter. Similarly, Firat (2022*b*) demonstrate that the wage Phillips curve exhibits non-linearity, especially in heated labor markets. Additionally, Benigno and Eggertsson (2024) identify a slanted L-shaped Phillips curve, which becomes steeper in tight markets and flatter as slack increases.

To capture nonlinearity in a flexible way, this research assumes that inflation is a cubic function of the slack variable. It investigates the V/U ratio's nonlinear impact on inflation, showing the importance of quadratic and cubic terms in modeling how inflation responds to labor market slack. [3]

Studies show slack measures have linear effects, but a cubic functional form, being more flexible, encompasses a linear specification as a special case. If slack is truly linear, coefficients for quadratic and cubic terms should be near zero, with non-significant p-values. Thus, using a nonlinear specification for all slack measures ensures a fair comparison by avoiding bias and unequal advantages.

3.1 OLS Analysis: Fixed Natural Rates

Regression Specification. I now present the primary regression model for aggregate level analysis, adopting cubic function approach to capture potential nonlinear relation-

[2]Notably, Bolhuis, Cramer and Summers (2022) mentions that the BLS's 1981 methodology change for estimating homeownership costs to owner's equivalent rent, moving from housing prices and mortgage costs to rents, could substantially affect two CPI series' comparability. To align inflation data comparability and wage inflation data availability, this analysis begins in 1985, following Ball, Leigh and Mishra (2022).

[3]In the appendix, the analysis highlights the importance of incorporating quadratic and cubic terms in the aggregate-level Phillips curve equation. This is confirmed by a Wald test on these terms for both the shorter and full sample periods.

ships between labor market slack and inflation:

$$\pi_t - \pi_t^e = \beta_0 + \beta_1 S_t + \beta_2 S_t^2 + \beta_3 S_t^3 + \epsilon_t, \tag{2}$$

where π_t is the quarterly seasonally adjusted annualized Median CPI inflation rate; [4] π^e represents 10-year-ahead inflation expectations from the Survey of Professional Forecasters (SPF); [5] the slack term, S_t, is calculated as a four-quarter average of either the unemployment rate or the V/U ratio.

Estimates. Beginning with the findings, I estimate six variations of the primary equation (2): two measures of slack (unemployment rate and V/U ratio, with an assumed constant natural rate) are analyzed separately, and both slack measures are included in the same regression for a direct "horse race" comparison, across two sample periods (1985 to 2019, and 1985 to 2023).

In table 1, columns 1 to 3 present the pre-pandemic sample results, while columns 4 to 6 showcase the full sample findings.

The analysis reveals the following conclusions. Pre-pandemic data analysis in columns 1 and 2 shows both the unemployment rate and V/U ratio as significant, with comparable adjusted r-square values indicating a similar fit. Incorporating both slack measures into one regression for the pre-pandemic period (column 3) shows they are equally effective in explaining inflation, as indicated by their p-values. Since both measures are significant at the 1-percent level, determining a superior measure is challenging, especially when one does not seem to dominate the other. [6]

Analyzing the full sample period results from columns 4 to 6 reveals the V/U ratio's superiority over the longer sample. It achieves an adjusted r-squared nearly double that of the unemployment rate, demonstrating a stronger fit, especially when including pandemic-era observations. Despite both measures showing significance at the 1-percent level individually, a "horse race" comparison reveals a clear difference. The p-value for testing the joint significance of linear, quadratic, and cubic terms is nearly zero for the V/U ratio, while the F-statistic becomes notably insignificant for the unemployment rate. The variation in the V/U ratio explains the surge in inflation during the pandemic years more effectively than that in the unemployment rate.

In summary, using aggregate national time series, this study validates the caution

[4] The appendix presents additional results using different measures of core inflation.

[5] Utilizing the extended dataset by Ball, Leigh and Mishra (2022).

[6] Column 1 indicates the unemployment rate terms are not individually significant, but jointly they are. This stems from collinearities.

	1985Q1-2019Q4			1985Q1-2023Q4		
	(1) U	(2) V/U	(3) Horse Race	(4) U	(5) V/U	(6) Horse Race
Unemp. Rate	-1.24		-4.70	-6.54**		3.93
	(1.71)		(2.97)	(3.26)		(3.34)
U-squared	0.14		0.55	0.91*		-0.58
	(0.28)		(0.44)	(0.48)		(0.48)
U-cubed	-0.01		-0.02	-0.04*		0.03
	(0.01)		(0.02)	(0.02)		(0.02)
V/U		9.84**	21.42***		6.27**	2.94
		(4.11)	(6.01)		(2.90)	(4.07)
(V/U)-squared		-11.54*	-28.70***		-6.14*	-2.40
		(6.06)	(8.51)		(3.50)	(5.30)
(V/U)-cubed		4.73*	11.38***		2.49**	1.38
		(2.76)	(3.58)		(1.16)	(1.73)
Constant	3.71	-2.67***	7.53	15.57**	-2.03***	-9.61
	(3.39)	(0.86)	(6.11)	(7.15)	(0.70)	(7.02)
R2	0.45	0.44	0.54	0.43	0.73	0.75
R2a	0.44	0.43	0.52	0.42	0.73	0.74
H0: U terms						
F-stat	15.41		5.46	7.46		1.05
P-value	0.00		0.00	0.00		0.37
H0: V/U terms						
F-stat		21.93	4.76		70.44	85.51
P-value		0.00	0.00		0.00	0.00
N	140	140	140	156	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1: Phillips Curve Estimation Results

Notes: Newey-West standard errors in parenthesis using a lag order of 4.

against adopting the V/U ratio based solely on pandemic years, suggesting potential cherry-picking. The findings reveal that V/U only shows its dominating superiority in the full sample period, including the pandemic years. Before the pandemic, both measures are equally effective, supported by similar and high significance levels. This reinforces the critique that the primary evidence for the switch does not stem from a single national time series.

3.2 Accounting for Time-Varying Natural Rates

This analysis moves from fixed to time-varying natural rates at the national level. Recognizing that the natural rate of unemployment may fluctuate over time suggests that the V/U ratio could also vary similarly. However, the Congressional Budget Office (CBO) provides time-varying rates for unemployment but does not offer analogous metrics for the V/U ratio. This study fills this gap by estimating time-varying natural rates for the V/U ratio through state-space models, accounting for the non-linearity in the Phillips curve. I estimate natural rates for both unemployment and the V/U ratio using a consistent methodology to ensure a fair comparison.

The Unscented Kalman Filter. Estimating natural rates using state-space models differs from the traditional Hodrick-Prescott (HP) filter, which assumes that the star variables increase directly with the actual series. In contrast, Bayesian filters incorporate inflation dynamics, thereby endogenously generating movements in the star variables. To address the equation's non-linearity, this study employs the Unscented Kalman Filter (UKF) for non-linear estimation, as developed by Wan and Van Der Merwe (2000).

The UKF Framework. The UKF setup with the Phillips curve framework can be written as a set of observation and transition equations. The observation equations are:

$$\pi_t - \pi_t^e = \beta_0 + \beta_1(S_t - S_t^*) + \beta_2(S_t - S_t^*)^2 + \beta_3(S_t - S_t^*)^3 + \epsilon_t^{PC} \quad (3)$$

$$S_t = S_t^* + (S_t - S_t^*) \quad (4)$$

Same as in the aggregate level regression framework, π_t represents the quarterly seasonally adjusted annualized median CPI inflation rate; π^e denotes the 10-year-ahead SPF inflation expectation; S_t is the 4-quarter average of the slack measure as specified in primary equation (2), and S_t^* is the to-be-established time-varying natural rate from the 4-quarter average of the slack measure. Equation 4 is used for technical identity purposes.

The time-varying natural rates of labor market slack are estimated by combining the observation equation, which includes inflation dynamics and slack variables in explicit gap form, with a set of transition equations:

$$S_t^* = S_{t-1}^* + \epsilon_t^{star} \tag{5}$$
$$S_t - S_t^* = \delta_1(S_{t-1} - S_{t-1}^*) + \epsilon_t^{gap}. \tag{6}$$

Equation 5 explicitly permits the natural rate of the slack variable S_t to vary over time. Following the literature, [7] this study models the star variable as following a random walk, where the error variance of ϵ_t^{star} indicates the amount of time variation. Equation 6 assumes the gap variable follows an AR(1) process.

The "Variance Ratio" Restriction. Kalman filter approaches in Phillips curve models enable simultaneous estimation of unknown parameters through maximum likelihood. However, as highlighted in works like Staiger, Stock and Watson (1997*b*), practical applications often restrict the star variables' variability by limiting the error variance in equation 5. Experiments in this study confirms that without restrictions, smoothed variables closely follow the actual series, becoming overly variable and conflicting with the natural rate concept. Therefore, applying a variance restriction on the star variable is necessary.

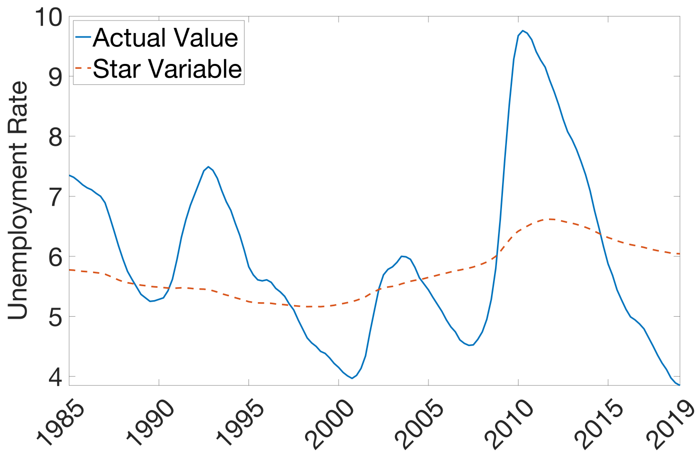
The next question concerns the appropriate level of variance restriction on the star variable. I use the CBO's natural rate as a benchmark for imposing this restriction. For the sample period from 1985Q1 to 2019Q4, the variance fraction of the CBO's natural rate to the raw unemployment rate's variance is about $8-9\%$. [8] When necessary, the Unscented Kalman Filter (UKF) applies an error-variance restriction of approximately 9% for both V/U and the unemployment rate. This adheres to both the CBO's benchmark and the guideline from Gordon (1997) regarding the degree of restriction: the natural rate should neither be completely flat nor excessively variable.

Estimates. Figure 3 illustrates the estimated time-varying natural rates for the unemployment rate and the V/U ratio from 1985 to the fourth quarter of 2019. It contrasts these smoothed series with the actual series, showing the movement of these natural rates.

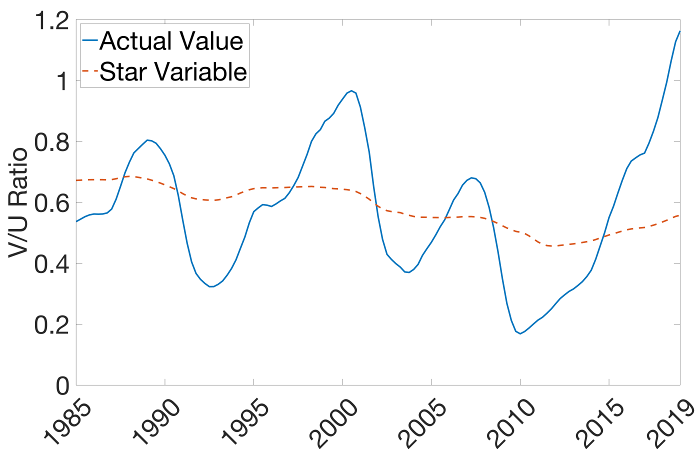
Furthermore, figure 4 extends this comparison to include data up to the fourth quarter

[7] Gordon (1997, 1998, 2011, 2013); OECD publishes natural rate estimates for many countries, detailed in works like Boone et al. (2003); Fabiani and Mestre (2004); Gianella et al. (2008); Guichard and Rusticelli (2011); Rusticelli (2014). Additionally, CBO outlines their labor market natural rate estimation approach in appendix B of Shackleton (2018).

[8] That is, var(nairu)/var(unemp. rate) is approximately 8 to 9 percent.



(a) U: Natural Rate vs. Actual Value

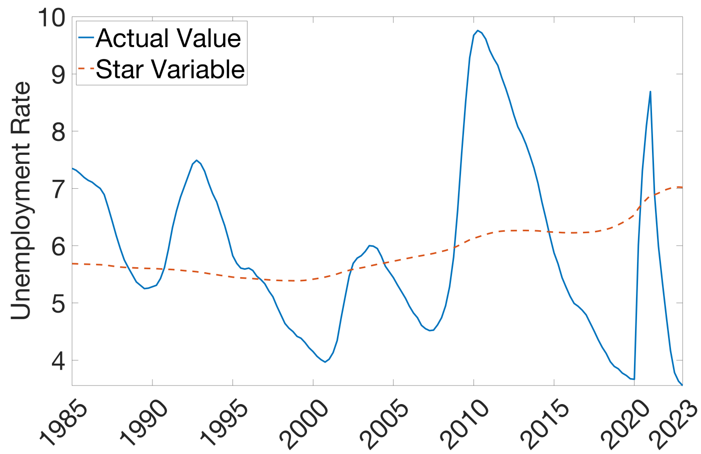


(b) V/U: Natural Rate vs. Actual Value

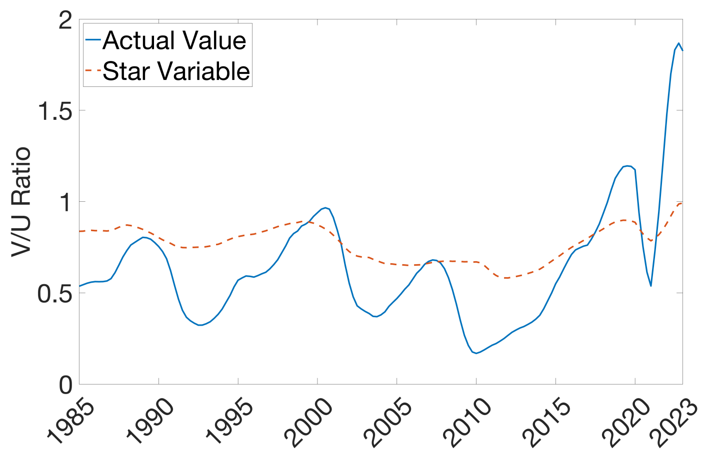
Figure 3: Price Inflation: UKF-Estimated Star Variables, 1985–2019Q4

of 2023. This period witnesses a notable rise in the estimated natural rate of unemployment during the pandemic. This increase is significant, as the early pandemic surge in unemployment did not correspond with a change in inflation, indicating that the natural rate adjustment helps the model explain such cyclical dynamics. As inflation began to rise, an elevated natural rate alongside a low unemployment rate suggests a significantly widened unemployment gap, indicating a tightened labor market. [9] However, even allowing the natural rate of unemployment to increase, the unemployment gap does not sufficiently explain inflation during the pandemic as table 2 later illustrates.

[9]This result aligns with Crump et al. (2022), who discovered that the actual natural unemployment rate might exceed CBO's estimates. Their research indicates the natural rate climbed to 5.9% by the end of 2021, with an unemployment gap of -1.5%.



(a) U: Natural Rate vs. Actual Value



(b) V/U: Natural Rate vs. Actual Value

Figure 4: Price Inflation: UKF-Estimated Star Variables, 1985–2023Q4

Moving to the estimation results, table 2 reports findings from the UKF estimation for two sample periods. Columns 1 and 2 correspond to the pre-pandemic period, while columns 3 and 4 present estimates from 1985 to 2023.

The aggregate analysis comprises two comparisons: explanatory power and p-values. Starting with pre-pandemic findings, the study confirms the joint significance of V/U's non-linear terms, supporting the use of a cubic form to model non-linearity. The Wald statistic reveals both the unemployment rate and V/U ratio terms are equally significant at nearly a 0-percent level, with small differences. Both metrics exhibit similar adjusted r-squared values, indicating comparable data fit.

Moving to the full sample period, the V/U model outperforms the unemployment rate model, as shown by the adjusted r-squared values and the joint significance of the

	1985Q1-2019Q4		1985Q1-2023Q4	
	(1) Unemp. Rate	(2) V/U Ratio	(3) Unemp. Rate	(4) V/U Ratio
motion eq.				
$(S - S^*)_{t-1}$	0.99***	0.99***	0.97***	0.99***
	(0.02)	(0.02)	(0.02)	(0.02)
measurement eq.				
slack gap	-0.31***	1.56***	-0.35***	1.94***
	(0.05)	(0.35)	(0.08)	(0.25)
slack gap^2	-0.03	-7.21***	0.06***	-0.91**
	(0.03)	(1.11)	(0.01)	(0.45)
slack gap^3	-0.00	9.63***	-0.02***	4.44***
	(0.01)	(2.92)	(0.01)	(0.75)
constant	0.04	0.18***	-0.01	0.41***
	(0.05)	(0.05)	(0.09)	(0.06)
error variances				
S^* eq.	0.00420	0.00011	0.00327	0.00048
	(restricted)	(restricted)	(restricted)	(restricted)
$S - S^*$ eq.	0.05***	0.00***	0.13***	0.00***
	(0.00)	(0.00)	(0.01)	(0.00)
PC eq.	0.19***	0.17***	0.44***	0.21***
	(0.02)	(0.02)	(0.04)	(0.03)
$H_0 : \beta_2 = \beta_3 = 0$				
Wald-stat	2.47	43.34	34.91	49.33
P-value	(0.29)	(0.00)	(0.00)	(0.00)
$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$				
Wald-stat	184.01	158.11	314.32	848.26
P-value	(0.00)	(0.00)	(0.00)	(0.00)
R2a	0.52	0.58	0.60	0.82
N	140	140	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The error variance of the star variables is restricted to approximately (for numerical stability) 9 percent of total variance, i.e., $var(star)/var(slack) \approx 0.09$.

In particular, in first subsample, $var(ur*)/var(ur) \approx 0.09$, and $var((v/u)*)/var(v/u) \approx 0.09$.

In full sample, $var(ur*)/var(ur) \approx 0.09$, and $var((v/u)*)/var(v/u) \approx 0.09$.

Table 2: Phillips Curve Estimation Results with Time-varying Natural Rates

slack measures' linear, quadratic, and cubic terms. Including pandemic observations enhances the joint significance Wald statistic nearly fourfold, favoring the V/U model. Moreover, the adjusted r-square value for the unemployment rate model suggests that the increase in the natural rate cannot fully explain the rise in inflation during the pandemic years.

This study confirms that analyses using both fixed and time-varying natural rates at the national level provide consistent outcomes, revealing a tie between both measures. However, there is a trend in recent literature towards using regional business cycle data as a more comprehensive source of evidence beyond just one single episode. An important finding, discussed in the next section, is how regional analysis supports the V/U ratio's superiority based on broader evidence.

4 V/U vs. U: Regional Evidence

I now turn to the regional analysis of the price Phillips curve. A central question raised by Mavroeidis, Plagborg-Møller and Stock (2014) concerns the specification uncertainty and limited information provided by national time-series data.

Panel data analysis offers greater variation due to the presence of multiple local business cycles in different regions. By using data from various regions, this method overcomes the problem of insufficient variation inherent in a single national time series, which may be influenced by a single event.

Additionally, employing fixed effects addresses potential error correlation issues that could arise from the response of monetary policy to the business cycle, or from expectations terms correlated with current economic conditions. [10] Assuming that these omitted variables are functions of either time or entity fixed effects, the use of two-way fixed effects in a panel setup can absorb the natural rates of slack measures, as well as mitigate concerns about how to measure inflation expectations. [11]

Considering these reasons, the empirical Phillips curve literature has shifted towards cross-sectional data, prompting this study to analyze the Phillips curve at the regional level. This section presents the regional econometric specification and the results.

Regional Data. For the price Phillips curve analysis, I use data on inflation rates in the 18 largest Metropolitan Statistical Areas (MSAs). The MSA-level Job Openings and Labor Turnover Survey (JOLTS), published as a one-time research release, provides quarterly data from 2000 to 2019 but has not been extended by the Bureau of Labor Statistics (BLS) to include the pandemic period. Therefore, I fit the Phillips curve using data from these 18 MSAs (and 13 MSAs for the time-varying natural rate analysis), [12] constraining the

[10] Single time-series data face endogeneity issues. Cost-push shocks are difficult to account for, and monetary policy responds to the business cycle. McLeay and Tenreyro (2020) argue that directly controlling for cost-push shocks is challenging because the primary drivers of these shocks can vary across periods. They also point out the problem of endogeneity with monetary policy, as central banks often attempt to optimally offset aggregate demand shocks.

[11] Inflation expectations, particularly those held by price and wage setters, are key driving variables in the Phillips curve but are difficult to measure. D'Acunto, Malmendier and Weber (2022) provide a comprehensive review of current inflation expectation surveys, noting potential measurement errors, including those associated with the commonly used Survey of Professional Forecasters in aggregate Phillips curve analysis.

[12] In the appendix, a full list of the 18 largest areas is provided. Following the approach of Babb and Detmeister (2017) and McLeay and Tenreyro (2020), these 18 largest areas cover approximately 30 percent of the civilian nonfarm labor force.

For the time-varying natural rate analysis, 13 MSAs are included, omitting 5 MSAs due to their limited data of only 6 observations.

analysis to the pre-COVID sample period. The dependent variable is the quarterly annualized Consumer Price Index (CPI) inflation rate, excluding food and energy, seasonally adjusted using the X13 algorithm.

4.1 OLS Analysis: Fixed Natural Rates

Regional Regression Specification. For the cross-sectional panel regression of price inflation, I estimate a cubic equation for quarterly CPI excluding food and energy (CPIXFE) inflation:

$$\pi_{it} = \beta_1^R \left[\frac{1}{4} \sum_{j=0}^{3} S_{i,t-j} \right] + \beta_2^R \left[\frac{1}{4} \sum_{j=0}^{3} S_{i,t-j} \right]^2 + \beta_3^R \left[\frac{1}{4} \sum_{j=0}^{3} S_{i,t-j} \right]^3 + \beta_4^R \pi_{i,t-1} + \alpha_i + \delta_t + \epsilon_{it}, \tag{7}$$

where π_{it} is the quarterly annualized inflation rate in region i at time t. The slack variable, $S_{i,t-j}$, is averaged over four quarters, similar to the aggregate Phillips curve analysis. $\pi_{i,t-1}$ is the lagged inflation rate. α_i and δ_t represent entity and time fixed effects, respectively, to adjust for omitted variable bias.

In this econometric analysis, employing two-way fixed effects is crucial for addressing endogeneity. Entity fixed effects mitigate biases from variables constant over time but varying across regions, such as productivity growth differences among different regions. For instance, if Maryland consistently shows a different slack measure level compared to California, entity fixed effects adjust for this. Time fixed effects control for characteristics uniform across regions but varying over time, such as nationwide monetary policy or long-run inflation expectations. Essentially, omitted variable bias is addressed for any variables that can be accounted for through time or entity fixed effects.

Regional Estimates. For the period from 2000 to before the pandemic, Table 3 presents three variations of the regional regression specification. Column 1 uses the unemployment rate, column 2 uses the V/U ratio as the slack measure, and column 3 includes both variables in a joint regression to evaluate their effectiveness through p-value comparison.

Both slack measures perform similarly in individual regressions, showing comparable within-R^2 values and significant p-values. However, when both variables are included together in column 3, the V/U ratio demonstrates greater relative efficacy. Specifically, the V/U ratio is significant at the 5-percent level (p-value $= 0.02$), while the unemployment rate is only weakly significant at the 10-percent level (p-value $= 0.08$).

From the analysis in Table 3, it is evident that incorporating time and entity fixed

	CPIX: 2001Q4-2019Q4		
	(1) U	(2) V/U	(3) Horse Race
Unemp. Rate	-1.68***		-1.45*
	(0.44)		(0.82)
U-squared	0.13**		0.15*
	(0.05)		(0.09)
U-cubed	-0.00*		-0.01*
	(0.00)		(0.00)
V/U		9.59***	6.47***
		(1.36)	(2.41)
(V/U)-squared		-5.31***	-4.09***
		(0.97)	(1.39)
(V/U)-cubed		1.05***	0.85***
		(0.22)	(0.28)
Lagged inf.	0.01	0.02	0.01
	(0.06)	(0.05)	(0.05)
Time FE	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes
R2a	0.55	0.55	0.55
R2-within	0.06	0.06	0.07
H0: U terms			
F-stat	12.75		2.36
P-value	0.00		0.08
H0: V/U terms			
F-stat		20.70	3.33
P-value		0.00	0.02
N	979	979	979

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: MSA, Price PC

Notes: Driscoll-Kraay standard errors in parenthesis using a lag order of 4.

effects, and using CPIXFE to measure price inflation, the V/U ratio demonstrates greater identification power. It prevails in the p-value comparison even in the pre-pandemic period.

4.2 Accounting for Time-Varying Natural Rates

Next, I extend the regional analysis to include time-varying natural rates. The main difficulty lies in estimating each region's unique natural rate along with all the model parameters within a cubic Phillips curve framework. To address this challenge, this paper employs the Extended Kalman Filter (EKF). The EKF uses the same state-space model but is computationally more efficient due to log-linearization.

The Extended Kalman Filter Framework. Similar to the UKF framework, the Extended Kalman Filter has both transition equations and observation equations.

For a single region i, the Phillips curve observation equation is as follows:

$$\pi_{it} - \pi_t^e = \beta_0 + \beta_1^R (S_{i,t} - S_{i,t}^*) + \beta_2^R (S_{i,t} - S_{i,t}^*)^2 + \beta_3^R (S_{i,t} - S_{i,t}^*)^3 + \beta_4^R (\pi_{i,t-1} - \pi_t^e) + \epsilon_{it}, \tag{8}$$

where π_{it} represents the quarterly annualized CPIXFE inflation rates; $S_{i,t}$ is the four-quarter average of slack measures; $S_{i,t}^*$ denotes the four-quarter average natural rate for specific region i; $\pi_{i,t-1}$ is the one-period lagged inflation, and π_t^e is the contemporaneous SPF-10 year ahead inflation expectation. [13] The identity equation is omitted for simplicity.

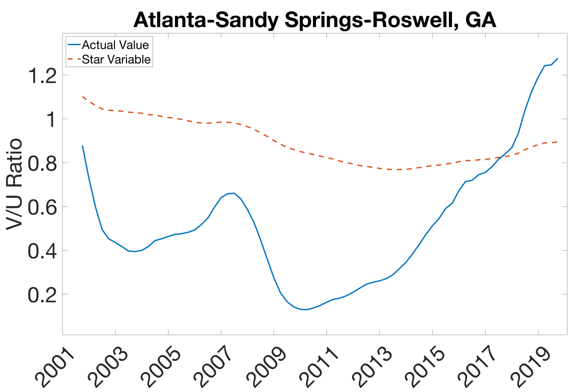
$$S_{it}^* = S_{i,t-1}^* + \epsilon_{it}^{star} \tag{9}$$

$$S_{it} - S_{it}^* = \delta_1 (S_{i,t-1} - S_{i,t-1}^*) + \epsilon_{it}^{gap}. \tag{10}$$

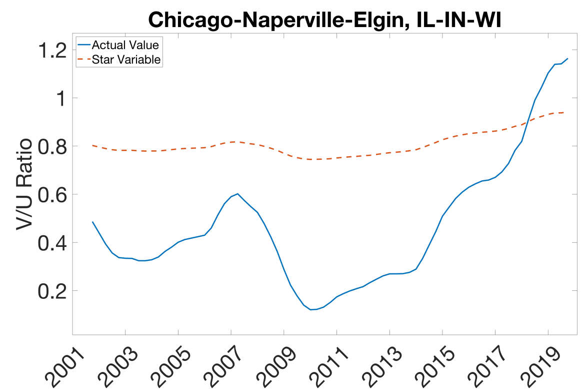
For the transition equations (9) and (10), the setup mirrors that of the UKF framework. The natural rates follow a random walk, and the gap adheres to an AR(1) process. The "variance ratio" restriction is also imposed on the star variable, following the rationale applied at the national level.

[13] The PC specification effectively assigns the coefficient of lagged inflation as α, and that of expected inflation as $1 - \alpha$. This ensures that the coefficients for expected and lagged inflation sum to one. The underlying theory implies that in a regime where everyone expects 2% inflation and the economy is in a steady state, it should indeed produce 2% inflation.

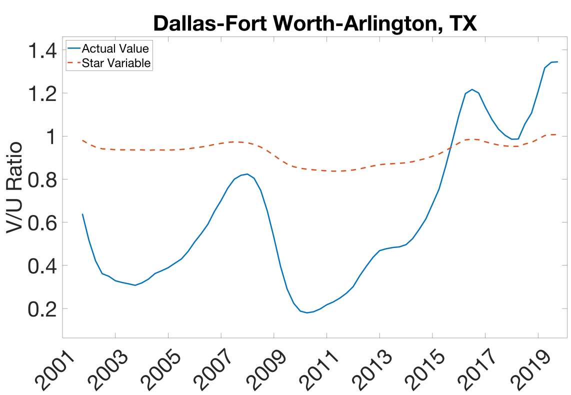
Estimates. Figure 5 and 6 display the estimated star variables for the first four MSAs in the sample. Given the similarity across figures, additional representations are provided in the appendix. These figures illustrate the time-path of the star variables, with estimates from the EKF appearing reasonable, as the natural rate of unemployment and V/U ratios are within plausible ranges.



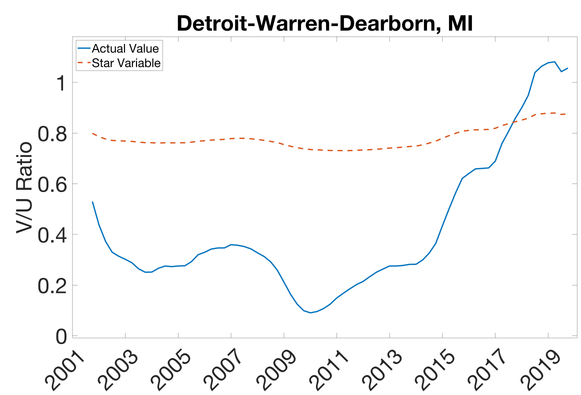
(a) Natural Rate of V/U - Atlanta



(b) Natural Rate of V/U - Chicago



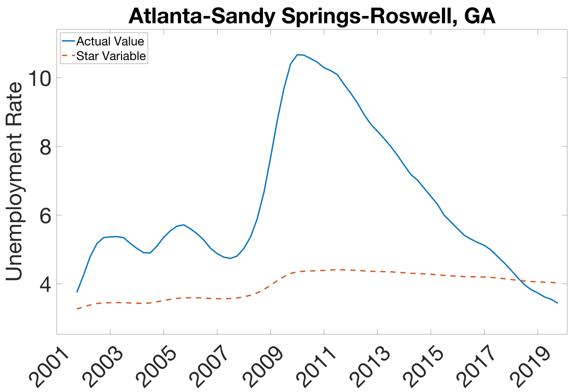
(c) Natural Rate of V/U - Dallas



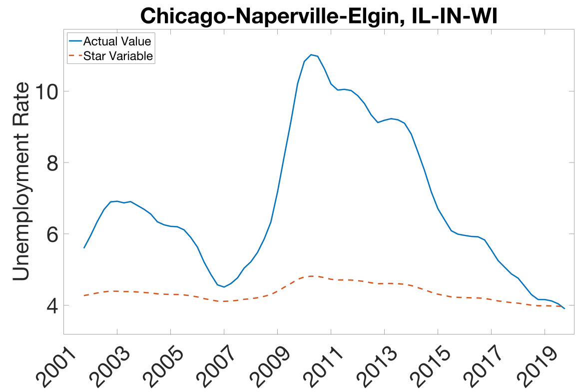
(d) Natural Rate of V/U - Detroit

Figure 5: Natural Rate vs. Actual V/U Values for MSAs: Atlanta, Chicago, Dallas, Detroit

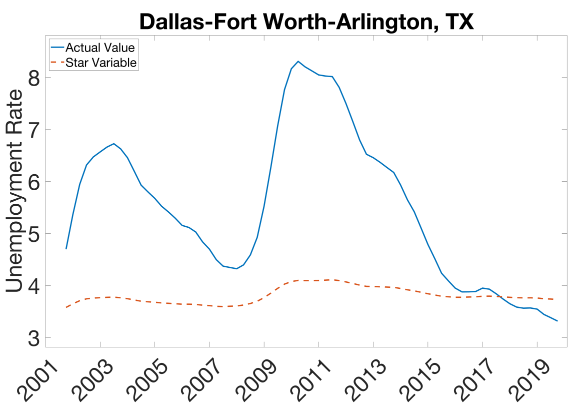
The model incorporates a cubic Phillips curve observation equation that enables economic slack to impact inflation. The corresponding fitted inflation figures, which also appear reasonable, are included in the appendix.



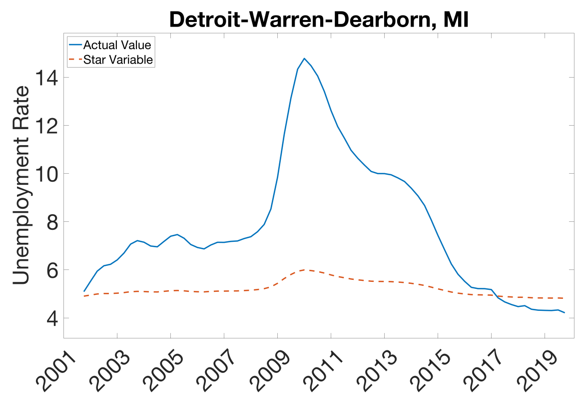
(a) Natural Rate of U - Atlanta



(b) Natural Rate of U - Chicago



(c) Natural Rate of U - Dallas



(d) Natural Rate of U - Detroit

Figure 6: Natural Rate vs. Actual U Values for MSAs: Atlanta, Chicago, Dallas, Detroit

Turning to the estimated Phillips curve coefficients in the EKF framework, table 4 presents the maximum likelihood estimates. Although the AR(1) coefficient appears high, these estimates yield plausible natural rate series. This may stem from the necessity of imposing variance restrictions on the star variable, suggesting that while the natural rate should not move very much, the gap has to persist a bit. Overall, the estimates generate reasonable series for natural rates.

	2001Q4-2019Q4, Extended Kalman Filter	
	(1) Unemp. rate model	(2) V/U model
motion eq.		
$(S-S^*)_{t-1}$	0.99***	0.99***
	(0.00)	(0.00)
measurement eq.		
slack gap	-0.38***	1.27***
	(0.06)	(0.34)
slack gap^2	0.04	-2.83***
	(0.03)	(0.56)
slack gap^3	-0.00	1.59*
	(0.97)	(0.00)
lagged inf.	-0.21***	-0.21***
	(0.03)	(0.03)
constant	-0.12	0.22**
	(0.10)	(0.09)
error variances		
S^* eq.	0.0105	0.0004
	(restricted)	(restricted)
$S-S^*$ eq.	0.09***	0.00***
	(0.00)	(0.00)
PC eq.	4.28***	4.22***
	(0.18)	(0.20)
$H_0: \beta_2 = \beta_3 = 0$		
Wald-stat	2.29	44.44
P-value	(0.32)	(0.00)
$H_0: \beta_1 = \beta_2 = \beta_3 = 0$		
Wald-stat	72.44	173.77
P-value	(0.00)	(0.00)
Vuong-stat	-33.37	
P-value	0.00	
R2a	0.12	0.13
R2-within	0.12	0.13
Log likelihood	-1467.66	192.85
N	949	949

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The Vuong test statistic: We report Vuong's 2-step test for overlapping models, testing the unemployment rate model against V/U model.

Step 1 Test Result: p-value for variance test is 0.00, where first stage test statistic is 2476.01, and 95-percentile of weighted sums of chi-square is 6.48.

The Wald test statistic: We report Wald test statistic for the joint significance of linear, quadratic, and cubic terms, and thus $W \sim \chi^2_{q=3}$.

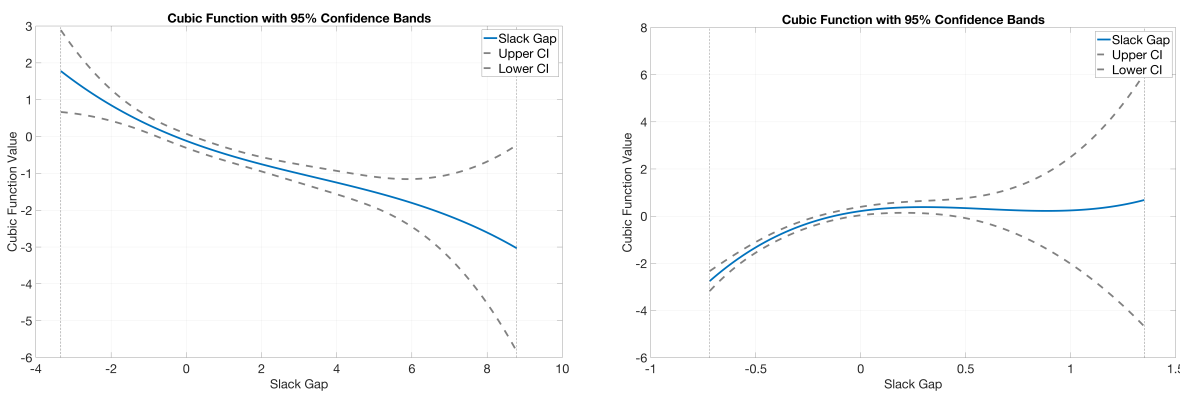
Table 4: EKF, Price PC, restriction on star variable error-variance only

Analyzing the fit, the within-R2 for the V/U ratio is slightly higher. Both measures show significant nonlinearity, as evidenced by the Wald test, aligning with past studies. All slack terms are jointly significant, with V/U showing a notably higher statistic.

For model selection within the Kalman Filter framework, log likelihood values for both models enable a comprehensive model selection test, enriching the analysis beyond mere p-value comparison. Vuong (1989) proposes a test based on log likelihoods and provides a distribution theory for the log likelihood-based test statistic, similar to the LR statistic.

Large positive values indicate a preference for the unemployment rate model, while large negative values favor the V/U model. This test determines if one model is closer to reality, with negative significant results favoring the V/U model. Given the large sample size in the regional analysis, the Vuong test yields a significant negative result, indicating the V/U model's superiority.

To examine the Phillips curve coefficients more closely, I illustrate the function's shape using the estimated coefficients. Figure 7 plots the cubic functions for both the unemployment rate and V/U ratio, including standard error bands. These curves align with expectations: downward for the unemployment rate and upward for the V/U ratio.



(a) Based on U PC Estimates

(b) Based on V/U PC Estimates

Figure 7: Cubic Function, 1985-2019Q4

5 Wage Inflation

This section delves into the wage inflation Phillips curve analysis, primarily examining the pre-pandemic period with a focus on fixed natural rates analysis, with further discussions deferred to the appendix.

Price inflation, crucial for consumers and central banks, plays a significant role in monetary policy. However, wage inflation also holds significance, affecting firm costs and, consequently, price levels. Many empirical and theoretical work has been investigating the pass-through effect from wages to prices.

5.1 Aggregate Analysis

OLS Regression Specification. To more thoroughly investigate wage inflation behavior, I apply the same cubic analysis used in the price Phillips curve, but focusing on the wage inflation gap instead of the median price inflation gap. The regression model incorporates linear, quadratic, and cubic functions of the slack variable, and, consistent with prior research on wage Phillips curve, includes a term for trend productivity growth.

The econometric model is as follows:

$$\pi_t - \pi_t^e = \beta_0 + \beta_1 S_t + \beta_2 S_t^2 + \beta_3 S_t^3 + \beta_4 LaborProductivity + \epsilon_t. \tag{11}$$

In this analysis, the inflation measure is the growth rate of the Employment Cost Index (ECI), which accounts for shifts in the composition of the labor force. The labor productivity term is measured as follows. I take the logarithm of the output per hour in the non-farm sector. Then, I apply the Hodrick-Prescott filter with a smoothing parameter of 16,000 to extract the trend component. Productivity growth is calculated as 400 times the quarterly change in this trend component. The definitions of the other variables remain consistent with earlier price behavior analyses.

Estimates. For both sample periods, from 1985 to 2019 and extending to 2022, table 5 indicates consistent insights with those found in the price Phillips curve analysis.

In columns 1 and 2, for the pre-pandemic sample, both indicators fit the data equally well in a cubic relationship. Individual regressions of the wage inflation gap on the cubic functions of the slack measures respectively yield p-values indicating nearly 0-percent significance. Column 3's p-value "horse race" shows both measures are equally significant. With both achieving nearly 0-percent statistical significance, a tie is a more reasonable conclusion than declaring a winner.

	1985Q1-2019Q4			1985Q1-2023Q4		
	(1) U	(2) V/U	(3) Horse Race	(4) U	(5) V/U	(6) Horse Race
Unemp. Rate	-0.78		-4.39**	-2.80		1.38
	(1.41)		(1.77)	(1.80)		(2.77)
U-squared	-0.01		0.49*	0.29		-0.24
	(0.22)		(0.26)	(0.29)		(0.38)
U-cubed	0.00		-0.02	-0.01		0.01
	(0.01)		(0.01)	(0.01)		(0.02)
V/U		-1.15	8.82		1.92	5.99
		(2.22)	(5.85)		(1.78)	(4.22)
(V/U)-squared		6.02*	-9.30		0.55	-3.46
		(3.49)	(7.60)		(2.28)	(5.05)
(V/U)-cubed		-3.33**	2.50		-0.30	0.92
		(1.65)	(3.15)		(0.79)	(1.66)
Labor Prod.	0.30***	0.59***	0.34***	0.15	0.54***	0.60***
	(0.08)	(0.09)	(0.12)	(0.15)	(0.11)	(0.15)
Constant	3.44	-1.80***	9.11**	8.21**	-2.18***	-6.18
	(2.86)	(0.33)	(3.93)	(3.51)	(0.37)	(6.55)
R2	0.54	0.51	0.56	0.45	0.55	0.57
R2a	0.52	0.50	0.54	0.43	0.54	0.55
H0: U terms						
F-stat	57.84		9.54	20.44		2.04
P-value	0.00		0.00	0.00		0.11
H0: V/U terms						
F-stat		107.68	5.49		172.32	11.98
P-value		0.00	0.00		0.00	0.00
N	140	140	140	156	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: National, Wage PC

Notes: OLS and Newey-West standard errors in parenthesis using a lag order of 4.

For the full sample (column 4 though 6), the findings mirror those from the price analysis: during the pandemic period, the unemployment rate is outperformed by the V/U ratio. In column 5, the V/U ratio model shows moderately higher explanatory power, with an R^2 of 0.54 compared to 0.43 for the unemployment rate model reported in column 4. In the "horse race" comparison illustrated in column 6, the V/U model has a p-value close to zero, while the unemployment rate model has a p-value of 0.11. This result indicates that the V/U ratio has higher relative efficacy.

Accounting for Time-Varying Natural Rates. I also consider the case of a time-varying natural rate for the wage Phillips curve, with results presented in the appendix. In this analysis, I employ the UKF framework to jointly estimate natural rates within the cubic specification of the wage Phillips curve while controlling for productivity growth. Overall, the UKF estimates corroborate the findings based on assuming a fixed natural rate. They indicate that in the pre-pandemic sample, both slack measures perform equally well. However, when the pandemic period observations are included, the V/U ratio outperforms the unemployment rate.

5.2 Regional Analysis

At the regional level, I use quarterly wage inflation rates derived from mean weekly wages reported by the Quarterly Census of Employment and Wages (QCEW) program. To analyze the data, I generate a residualized scatter plot, shown in Figure 8.

The vertical axis displays residuals from regressing quarterly annualized wage inflation against state dummies, time dummies, and the lagged inflation term. The horizontal axis shows residuals from regressing the unemployment rate or the V/U ratio against state, time dummies, and the lagged inflation term. I then plot the residualized wage inflation against the residualized slack terms. The data appear somewhat noisy.

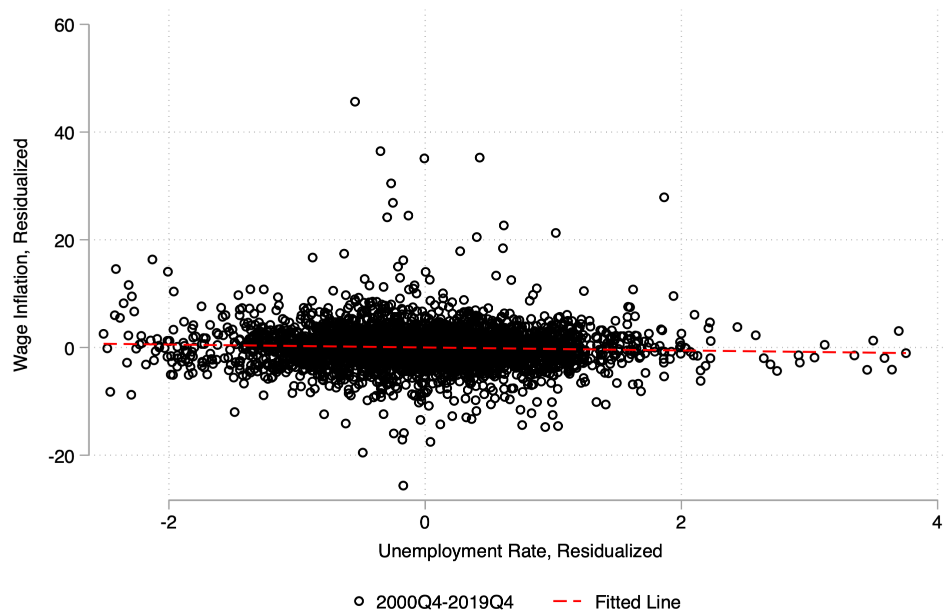
Panel Regression Estimates. Table 6 reports the pre-pandemic estimates in columns 1 through 3 and the full sample estimates in columns 4 through 6. The within-R^2 values are similar across the two slack measures, but the F-statistics for the joint significance of the slack terms are higher for the V/U ratio model in the pre-pandemic period. Column 3 shows that the V/U ratio is jointly significant at the 5-percent level, while the unemployment rate is not statistically significant. When the analysis is extended to include the pandemic period observations, neither variable is statistically significant in the "horse race."

	2001Q3-2019Q4			2001Q3-2022Q4		
	(1) U	(2) V/U	(3) Horse Race	(4) U	(5) V/U	(6) Horse Race
Unemp. Rate	-0.63		-0.39	-1.13**		-0.48
	(0.49)		(1.01)	(0.51)		(0.82)
U-squared	0.03		0.07	0.09		0.05
	(0.07)		(0.11)	(0.08)		(0.10)
U-cubed	-0.00		-0.00	-0.00		-0.00
	(0.00)		(0.00)	(0.00)		(0.00)
V/U		7.05**	8.10*		5.62**	4.66
		(3.27)	(4.71)		(2.20)	(4.28)
(V/U)-squared		-3.81	-4.57		-2.70*	-2.40
		(2.57)	(3.21)		(1.54)	(2.33)
(V/U)-cubed		0.59	0.75		0.47	0.43
		(0.60)	(0.72)		(0.29)	(0.41)
Lagged Inf.	-0.36***	-0.36***	-0.36***	-0.33***	-0.33***	-0.33***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes
R2	0.57	0.58	0.58	0.63	0.63	0.63
R2a	0.56	0.56	0.56	0.62	0.62	0.62
R2-within	0.13	0.13	0.13	0.11	0.11	0.11
H0: U terms						
F-stat	2.45		0.26	3.99		0.11
P-value	0.07		0.85	0.01		0.95
H0: V/U terms						
F-stat		4.38	2.84		5.88	1.09
P-value		0.01	0.04		0.00	0.36
N	3774	3774	3774	4386	4386	4386

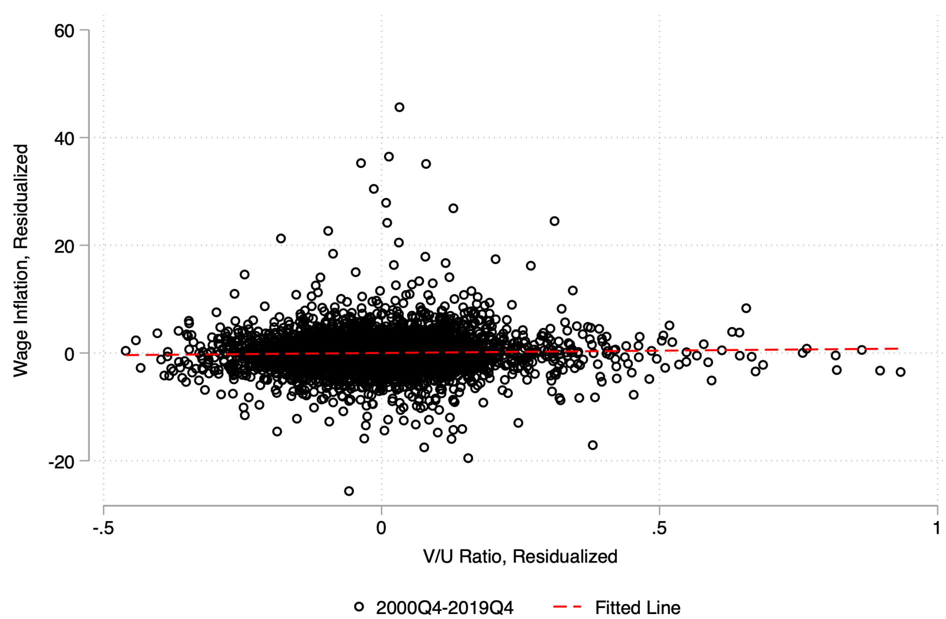
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: State, Wage PC

Notes: Driscoll-Kraay standard errors in parenthesis using a lag order of 4.



(a) Unemployment Rate



(b) V/U Ratio

Figure 8: Residualized Scatter Plots, State

6 A Simple Model of V/U and Inflation

This section illustrates why the V/U ratio is more appropriate for representing labor market tightness in the Phillips curve than the unemployment rate. To investigate short-run fluctuations around the long-run equilibrium, this section will adopt an approach different from the standard Diamond-Mortensen-Pissarides (DMP) model.

The canonical DMP model focuses on long-run determinants of employment and wages, considering factors such as the cost of creating vacancies and various characteristics of workers, which determine vacancy creation and wage conditions. Departing from the canonical model, I follow the tradition in short-run macroeconomics and assume that the demand determines the number of jobs J that firms would like to fill, thus eliminating one market-clearing condition for the supply of vacancies. This simplification allows the model to focus directly on how the economy behaves when J fluctuates exogenously, as opposed to being chosen given the number of vacancies.

Subsequently, the model-derived real wage is incorporated into a Phillips curve, drawing on the framework from Blanchard and Katz (1996) and Blanchard and Bernanke (2023), to present a straightforward inflation model by treating prices as a markup over wages. Furthermore, in the presence of shocks to the Beveridge curve, the curve can shift, making the relationship between inflation and U unstable, and the V/U ratio is a more accurate gauge.

6.1 V/U and the Wage Function

The Economy. Time is continuous and infinite. However, as the model intends to solve for a stationary equilibrium, the time subscript is dropped for simplicity. The economy consists of a fixed labor force, L, which is split between employed workers, E, and unemployed workers, U, such that

$$L = E + U.$$

The model accounts for short-term demand fluctuations. The exogenous variable J captures the number of jobs available in the economy, and J can be thought of as the total demand in the economy, in line with the ideas presented in Blanchard, Domash and Summers (2022). The exogenous changes in J then captures the change in aggregate demand for labor. [14] The economy can have one job for every person in the labor force, more jobs than the labor force, or fewer jobs than the labor force. [15]

[14] In this economy, J jobs are then matched with L workers.

[15] Blanchard, Domash and Summers (2022) assumes that increase in aggregate activity causes firms to

At any moment, there are E existing worker-job matches, with V vacancies posted by firms, and U amount of unemployed people. The model then has

$$J = E + V,$$

where a job can either be filled by a successful match or remain unfilled as a vacancy. Therefore, $J = E + V = L - U + V$.

The E existing worker-job matches are destroyed at an exogenous rate s, at which the workers separated from the job become unemployed and the job becomes vacant.

Workers and Firms. Agents have infinite lifespans. Workers are risk-neutral and identical, and thus can be matched with any job. They have the same marginal productivity, discounting the future at rate r. Employed workers earn a wage w, which is also the current utility when employed. For simplicity, unemployed workers receive no flow dividends, with unemployment benefits set to 0.

Similarly, firms have linear utility. [16] An employed worker generates a flow of output y, resulting in a per-period profit of $y - w$ for the firm. For simplicity, firms do not face a vacancy cost of c when searching for workers.

Firms and workers match through a function $H = m(U, V)$, utilizing Cobb-Douglas production technology. Additionally, there is a transition process between being employed and unemployed. The hiring process at any given time follows a Cobb-Douglas production model:

$$H = am(U, V) = aU^{\gamma}V^{1-\gamma},$$

where $a > 0$ and $\gamma \in (0, 1)$. At any moment, H represents the amount of job hiring. The model posits an equal probability of matching for each worker and firm. Other things being equal, an increase in jobs, driven by firms' desire to meet rising aggregate demand, results in more new hires given

$$H = am(U, J - L + U).$$

More jobs mean more vacancies looking to fill positions, affecting employment (E) and unemployment (U) rates while keeping the labor force (L) constant.

After a worker and a firm form a match, it is exposed a stochastic labor turnover

post more vacancies. That is, higher aggregate demand leads to greater number of vacancies, and lower level of unemployment.

[16] They are not necessarily homogeneous though. Given the model accounts for short run demand fluctuations, firms may possess some market power, potentially allowing them to charge a markup.

process characterized by an exogenous separation rate s, the Poisson arrival rate. Wages will result from Nash bargaining mechanism between the worker and the firm, where the worker's bargaining power is denoted by $\beta \in [0,1]$.

Labor Market Tightness and the Beveridge Curve. Labor market tightness is measured by $\theta = V/U$, where V and U represent aggregate measures of unmatched, posted vacancies and unemployed workers, respectively. This paper's approach to modeling labor market tightness aligns closely with the innovative aspect of the DMP framework. The primary distinction lies in how the two key hazard rates are influenced by exogenous variables L and J.

The number of hires H, labor market tightness θ, and unemployed U are related through the the *hazard rate of job finding*:

$$\textit{job finding rate} = \frac{H}{U} = \frac{a \cdot m(U,V)}{U} = aU^{\gamma-1}V^{1-\gamma} = a\theta^{1-\gamma}.$$

The daily probability of a searcher finding a job rises with increased labor market tightness, particularly when vacancies expand due to an exogenous rise in labor demand.

The number of hires H, labor market tightness θ, and vacancies V are linked through the *hazard rate of job filling*:

$$\textit{job filling rate} = \frac{H}{V} = \frac{a \cdot m(U,V)}{V} = aU^{\gamma}V^{-\gamma} = a\theta^{-\gamma}.$$

For the employer, the probability of filling a vacancy decreases, since as demand for labor increases, it is more challenging to fill vacancies, assuming the labor force remains constant.

Next, I calculate a steady-state level of vacancies as a function of the unemployment rate. The inverse relationship between vacancies and the unemployment rate defines the Beveridge curve. Traditionally, the unemployment rate is expressed as U/L, while the vacancy rate is typically V/J. For clarity, this study re-defines both rates relative to the labor force, L, using lowercase v and u to denote the vacancy and unemployment rates, respectively. Hence, θ can be defined as v/u.

At any given time, job separations amount to sE, while the number of job creations equals the job finding rate multiplied by U, or $a\theta^{1-\gamma}U$. In equilibrium, the flow from unemployment to employment equates the outflow from employment, the Beveridge curve relationship is explicitly:

$$\begin{aligned} s(E/L) &= \textit{job finding rate} \cdot (U/L) \\ u &= \frac{s}{a\theta^{1-\gamma} + s} \\ v &= a^{-\frac{1}{1-\gamma}} \cdot [s(1-u)]^{\frac{1}{1-\gamma}} \cdot u^{-\frac{\gamma}{1-\gamma}}. \end{aligned} \tag{12}$$

Search and Matching: Wage Determination. This section derives the wage as a function of labor market tightness, utilizing the Nash bargaining model of wage determination within the DMP framework. In this simplified model, all unemployed workers actively search for jobs without any job-to-job transitions. A match between a worker and a firm only ends when a job destruction shock occurs.

Although the model focuses on short-run dynamics, the subsequent derivation is a steady-state analysis that assumes rapid out-of-steady adjustments. This assumption is consistent with the observations of Blanchard, Domash and Summers (2022), who argue that adjustment dynamics are typically very fast, occurring within a few months at most. If the adjustment process is sufficiently quick, it is a reasonable approximation to assume that the economy is always in a steady state.

I examine workers' expected utility in different states: employed and unemployed. To find the steady state of the model, the value functions for workers in employment state, V^E, and unemployment state, V^U, meet the following Bellman equations:

$$r\mathbf{V}^E = w + s\left[\mathbf{V}^U - \mathbf{V}^E\right] \tag{13}$$

$$r\mathbf{V}^U = a\theta^{1-\gamma}\left[\mathbf{V}^E - \mathbf{V}^U\right] \tag{14}$$

The expected present discounted value of employing a worker for firms is represented as V^J, while the expected present discounted value for unmatched firms with a vacancy is V^V. The Bellman equations for firms are as follows:

$$r\mathbf{V}^J = y - w + s\left[\mathbf{V}^V - \mathbf{V}^J\right] \tag{15}$$

$$r\mathbf{V}^V = a\theta^{-\gamma}\left[\mathbf{V}^J - \mathbf{V}^V\right]. \tag{16}$$

A notable distinction between the canonical model and this paper's model is the absence of imposing a free-entry condition, meaning the value of a vacancy does not necessarily equate to zero. In the canonical model, firms are presumed to be competitive, resulting in vacancies having a value of zero at steady state.

However, in this model, particularly in the short term, the number of J is influenced by demand, and it is not essential to assume that firms are perfectly competitive. The firm's output is determined by demand, allowing it to produce and hire additional workers up to the limit set by aggregate demand. Essentially, firms aim to maximize sales, but their actual demand is capped at J.

For context, imagine each worker produces one unit of output, and the economy demands ten units. Consequently, firms aim to hire ten workers, and this requirement changes exogenously. Creating additional vacancies or hiring more workers would serve no purpose for firms if they cannot sell the extra output.

Solving the system of equations yields the following surplus values for workers and firms:

$$\mathbf{V}^E - \mathbf{V}^U = \frac{w}{(r + s + a\theta^{1-\gamma})} \tag{17}$$

$$\mathbf{V}^J - \mathbf{V}^V = \frac{y - w}{(r + s + a\theta^{-\gamma})} \tag{18}$$

This paper then derives the wage function relative to labor market tightness, based on the Nash bargaining model of wage determination within the DMP framework. The optimization problem is formulated as:

$$\max_w \left(\mathbf{V}^E - \mathbf{V}^U\right)^\beta \left(\mathbf{V}^J - \mathbf{V}^V\right)^{1-\beta},$$

where β represents a constant coefficient that measures bargaining power, independent of both parties' threat point values. [17]

Assuming workers and firms are risk-neutral with linear utility, this paper employs a surplus sharing solution to address the Nash bargaining problem, following Rogerson, Shimer and Wright (2005). Linear utility allows for transferable utility, enabling surplus to be transferred between workers and firms on a one-to-one basis in either direction. Hence, the Nash-bargained wage in a model with transferable utility is given by:

$$\mathbf{V}^E - \mathbf{V}^U = \beta \left(\mathbf{V}^E - \mathbf{V}^U + \mathbf{V}^J - \mathbf{V}^V\right)$$

$$w = \frac{\beta\left(r + s + a\theta^{1-\gamma}\right)}{(1-\beta)\left(r + s + a\theta^{-\gamma}\right) + \beta\left(r + s + a\theta^{1-\gamma}\right)} \cdot y, \tag{19}$$

[17] The worker's outside option is to remain unemployed, and the firm's outside option is to post a vacancy.

where the wage is determined by labor market tightness θ. [18]

The initial segment of the model focuses on how wages respond to changes in labor market conditions. If any of the exogenous variables change, for example, an increase in J, the number of available jobs, it leads to more hires for a given unemployment level and an increase in θ, which subsequently raises w.

The subsequent part of the model aims to integrate the real Nash-bargained wage into the wage and price Phillips curve.

6.2 V/U and the Phillips Curve

The first part of the model relates the exogenous changes in labor market tightness to the Nash-bargain wage, and the second part of the model links the real wage into the reduced form Phillips curves following the wage-price process in Blanchard and Katz (1996) and Blanchard and Bernanke (2023).

Wage Phillips Curve. First, the wage equation assumes that the nominal wage depends on price expectations for quarter t and a target real wage w^*. The target real wage is a weighted average of the Nash-bargained real wage, which is derived from the model as a function of labor market slack, and the realized real wage from the previous quarter, to account for inertia or slow adjustments in real wages.

The framework can be written as a system of equations:

$$w_t^{\text{nominal}} = p_t^e + w_t^*, \text{ where } w_t^* \text{ denotes target real wage} \tag{20}$$

$$w_t^* = \alpha \cdot w_t^{model}(\theta) + (1-\alpha) \cdot \left(w_{t-1}^{\text{nominal}} - p_{t-1} \right) \tag{21}$$

$$p_t = w_t^{\text{nominal}} + z_t^p \tag{22}$$

The last equation (22) implies that, in this simplified world, prices are essentially a markup over wages.

Labor market tightness plays a critical role in wage bargaining, with labor market conditions determining the real wage workers receive from surplus sharing. The model-determined wage then influences w_t^* through a coefficient α, which measures the extent to which the Nash-bargain wage translates into the target real wage.

By defining the nominal wage from equation (20) and incorporating w_t^* from equation (21), along with using equation (22) for the real wage, the reduced-form nominal wage

[18] The appendix demonstrates that with β positive and between 0 and 1, $\theta > 0$, and under the constant returns to scale (CRS) assumption that $(1-\gamma) > 0$, the wage function increases with θ.

Phillips curve is derived as follows: [19]

$$w_t^{\text{nominal}} = p_t^e + w_t^* \tag{23}$$
$$\Delta w_t^{\text{nominal}} = \pi_t^e + \alpha w_t^{\text{model}}(\theta) + \alpha z_{t-1}^p. \tag{24}$$

I define the inflation expectation for this period as $\Delta p_t^e = p_t^e - p_{t-1}$, representing the expected price level for this period minus the price level observed in the last period. It stays a variable to be measured, and I do not take a stance on how inflation expectation forms. For simplicity, I also ignore a shock term that might influence wage growth.

Price Phillips Curve. Transitioning from wages to prices, the model assumes that price is a markup over wages, suggesting a direct one-to-one impact from wages on prices:

$$p_t = w_t^{\text{nominal}} + z_t^p, \tag{25}$$

and thus price growth, defined as the first difference, is

$$p_t - p_{t-1} = (w_t^{\text{nominal}} - w_{t-1}^{\text{nominal}}) + (z_t^p - z_{t-1}^p) \tag{26}$$
$$\Delta p_t = \Delta w_t^{\text{nominal}} + \Delta z_t^p \tag{27}$$
$$\Delta p_t = \pi_t^e + \alpha w_t^{\text{model}}(\theta) + \epsilon_t^p. \tag{28}$$

Equation (28) outlines a standard expectations-augmented reduced form of the price Phillips curve, where the error term

$$\epsilon_t^p = z_t^p - (1-\alpha) z_{t-1}^p$$

follows an MA(1) process.

Equation (28) hints at why the relationship between the unemployment rate and inflation can become unstable during certain periods. The Beveridge curve can be rewritten as

$$u = \frac{s}{a\theta^{1-\gamma} + s}.$$

If matching efficiency a and the separation rate s are constant, there is a one-to-one rela-

[19] Note that $\alpha \cdot w_t^{\text{model}}$ in the cubic empirical specification will be

$$\alpha w_t^{\text{model}} = \alpha \left(\beta_1 Slack_t + \beta_2 Slack_t^2 + \beta_3 Slack_t^3 \right).$$

tionship between the unemployment rate u and labor market tightness θ. However, when the Beveridge curve shifts due to changes in a or s, the same unemployment rate u can correspond to different vacancy rates v and labor market tightness θ. Connecting this to the Phillips curve, these shocks to a and s cause the relationship between inflation and unemployment to become unstable.

Alternatively, consider the following log-linearization around steady-state values of u and v, where tilde variables represent percentage deviations as the economy adjusts to a different Beveridge curve. The detailed derivation is provided in Appendix D.2.3:

$$v = a^{-\frac{1}{1-\gamma}} \cdot [s(1-u)]^{\frac{1}{1-\gamma}} \cdot u^{-\frac{\gamma}{1-\gamma}} \tag{29}$$

$$\widetilde{v} = -\frac{1}{1-\gamma}\widetilde{a} + \frac{1}{1-\gamma}\widetilde{s} - \frac{1}{1-\gamma}\frac{u}{1-u}\widetilde{u} - \frac{\gamma}{1-\gamma}\widetilde{u} \tag{30}$$

$$\widetilde{\theta} = \widetilde{v} - \widetilde{u}. \tag{31}$$

Without shocks to the matching efficiency a or the separation rate s, $\widetilde{\theta}$ depends solely on $\widetilde{u}$. However, if there are shocks to a and s, the relationship between $\widetilde{u}$ and $\widetilde{v}$ changes, making the relationship between inflation and unemployment unstable.

6.3 Observed Shifts in the Beveridge Curve

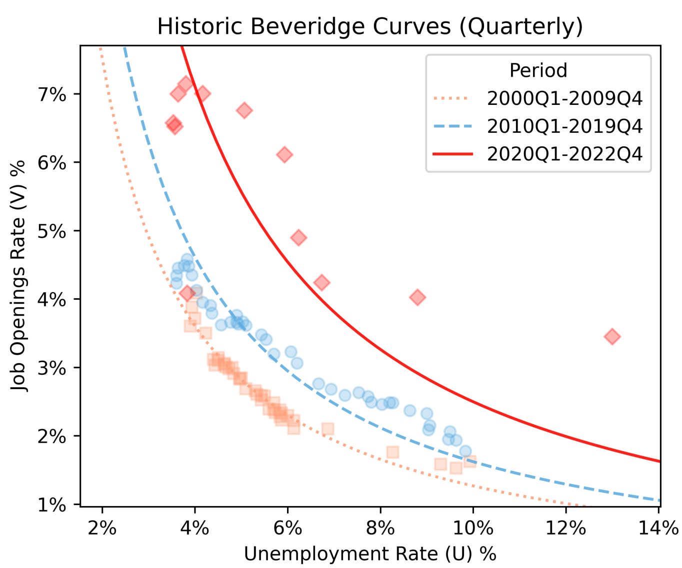


Figure 9: Historic Beveridge Curves, 2000Q1-2022Q4

To show the empirical relevance of the above shocks to the Beveridge curve, the following examines how the Beveridge curve behaved in the last 20 years in the United States. Figure 9 plots the job openings rate against unemployment rates from 2000 to 2022, with different markers and colors denoting various periods. It appears that there are three different periods, and it was stable till the 2009, and then it shifted out. Then, after the great recession, it shifted out again during the pandemic.

The model explains these shifts as resulting from changes in the matching efficiency parameter a and the separation rate s. An increase in a improves hiring at a given level of unemployment, lowering the unemployment rate for any fixed number of vacancies, thus shifting the Beveridge curve inward. Conversely, an increase in s shifts the Beveridge curve outward, similar to the effect of a decrease in a. I will then examine how the values of a and s may have changed.

To estimate the job separation rate, I follow the approach of Shimer (2005). Let U_t denote the number of unemployed workers at time t (where t refers to a monthly frequency) and let U_t^s represent the number of short-term unemployed workers. Assuming all unemployed workers find a job with probability f_t and none leave the labor force, the unemployment level at time $t+1$ is the sum of unemployed workers at time t who did not find a job and the newly unemployed at time $t+1$:

$$U_{t+1} = U_t \cdot (1 - f_t) + U_{t+1}^s.$$

From this equation, the job-finding rate f_t can be calculated.

For the job separation rate, Shimer (2005) shows that the number of short-term unemployed workers at time $t+1$ is approximately:

$$U_{t+1}^s = s_t E_t \cdot \left(1 - \frac{1}{2} f_t\right),$$

where E_t is the employment level at time t. The term $\left(1 - \frac{1}{2} f_t\right)$ accounts for the fact that, on average, a worker who loses the job has on average, half a month to find a new job before being recorded as unemployed (Shimer, 2005). Solving for the job separation rate s_t from the above equations gives:

$$s_t = \frac{U_{t+1}^s}{E_t \left(1 - \frac{1}{2} f_t\right)},$$

and s_t is later converted to a quarterly frequency.

Since the Beveridge curve appears stable within each period, I assume a constant a

for each period and estimate its value across different periods. Given the measures of s, u, and v, I estimate a using the Beveridge curve relationship in equation (29). Following the identified range for the elasticity with respect to the unemployment rate in Petrongolo and Pissarides (2001), [20] I set γ to 0.5. For each period, I estimate a by minimizing an error function. This error function is defined as the sum of squared errors between the observed vacancies and the predicted vacancies, where s represents the average separation rate for each period.

Table 7 reports the values for the three periods during which the Beveridge curve appears to be stable. It shows that the first outward shift in the Beveridge curve, around the 2010s, from the first period to the second period, was primarily due to a sharp decline in matching efficiency. Although this decline in a could have caused an even more significant shift, its impact was mitigated by a decrease in the job separation rate.

During the COVID era, the job separation rate increased, while matching efficiency continued to decline. These factors together led to a more salient outward shift, as they exacerbated each other's effects. [21]

Period	Estimated Value a	Separation Rate s
2000Q1-2009Q4	1.809	0.072
2010Q1-2019Q4	1.284	0.057
2020Q1-2022Q4	1.092	0.061

Table 7: Matching Efficiency and Separation Rate by Sample Period

Notes: The values reported in the table represent the estimated matching efficiencies a and the average separation rates for each period.

Recall that in 2021, the labor market was characterized by unemployment rates that were not particularly low but elevated values of V/U and a large, positive inflation gap, as shown in Figures 1 and 2. This observation is consistent with an outward shift in the Beveridge curve. For a constant unemployment U, there was a higher vacancy V, and thus a higher V/U. Therefore, the shift in the Beveridge curve during this period is one

[20] Petrongolo and Pissarides (2001) identified an empirical elasticity range for unemployment between 0.5 and 0.7.

[21] In this model framework, the shift can happen due to shocks to separation rate or matching efficiency. Recent studies (Barlevy et al., 2024; Blanchard, Domash and Summers, 2022) have explored underlying reasons for these shifts, identifying causes such as inflow rate changes and shocks to aggregate activity.
Blanchard, Domash and Summers (2022) conducts a similar analysis using different methodology. They develop a time-series for matching efficiency as $a = h/u^{\alpha}v^{1-\alpha}$, and variations in reallocation are calculated with h representing the ratio of gross hires to the labor force with α set at 0.4, spanning from January 2019 to April 2022.

of the key factors explaining why V/U suddenly began to behave differently from the unemployment rate, with the latter exhibiting an unstable relationship with inflation.

7 Conclusion

Previous literature has long used the unemployment rate as the go-to metric of labor market slack. However, the unusual combination of highly elevated inflation during the pandemic years, along with not particularly low unemployment rates, has sparked renewed debate on the right slack measure in the Phillips curve relationship.

Many recent studies based on national time-series data have found that the V/U ratio outperforms the unemployment rate in explaining inflation dynamics. Yet, Şahin (2022) criticizes that the divergence between the two measures during the COVID period was merely one episode, and changing the slack measure based on that single event can be dubious. This study contributes additional evidence that the V/U ratio is a more reliable measure of labor market slack than the unemployment rate, even in periods predating the pandemic years, using regional data.

First, I extend previous empirical work to include a time-varying natural rate in state-space models for both V/U and U. I also assume that inflation is a cubic function of the slack measure. With these extensions, I find that the two measures explain inflation dynamics equally well on national time-series data from the pre-pandemic periods. This finding confirms the critique that the V/U ratio's superiority may be specific to the pandemic period.

Second, this study leverages regional data. National data offer only a single episode to compare the two measures, but regional data provide more observations, effectively offering multiple episodes of evidence. Based on the regional data, this study finds that the V/U ratio outperforms the unemployment rate in the cubic specification of the Phillips curve, assuming a constant natural rate and incorporating two-way fixed effects. In an Extended Kalman Filter framework, I jointly estimate the natural rates of the two slack measures along with the Phillips curve. This approach also confirms the superiority of V/U even before the COVID period.

The paper then presents a simple framework to explain why V/U appears in the Phillips curve. As the labor market becomes tighter, workers' bargaining power increases, pushing up real wages. By incorporating this into a simple wage-price determination process, the resulting model yields an expectations-augmented Phillips curve with V/U included. It also shows that V/U matters because the Beveridge curve does not always remain in one place. It can shift permanently, and when it does, the same U can correspond to different V values; U alone cannot capture this information.

This work adds evidence supporting the V/U ratio as a more reliable measure of labor market slack, and it should play a larger role in economic forecasting and monetary policy. It also intuitively shows that shifts in the Beveridge curve are crucial for the unstable relationship between unemployment (U) and inflation. However, more research is needed to understand what determines the underlying parameters behind these shifts, such as shocks to matching efficiency and separation rates.

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APPENDIX

A Appendix: Aggregate Analysis

	1985Q1-2019Q4		1985Q1-2023Q4	
	(1)	(2)	(3)	(4)
	U	V/U	U	V/U
Unemp. Rate	-1.24		-6.54**	
	(1.71)		(3.26)	
U-squared	0.14		0.91*	
	(0.28)		(0.48)	
U-cubed	-0.01		-0.04*	
	(0.01)		(0.02)	
V/U		9.84**		6.27**
		(4.11)		(2.90)
(V/U)-squared		-11.54*		-6.14*
		(6.06)		(3.50)
(V/U)-cubed		4.73*		2.49**
		(2.76)		(1.16)
Constant	3.71	-2.67***	15.57**	-2.03***
	(3.39)	(0.86)	(7.15)	(0.70)
R2	0.45	0.44	0.43	0.73
R2a	0.44	0.43	0.42	0.73
H0: U Non-linear terms				
F-stat	0.29		1.88	
P-value	0.75		0.16	
H0: V/U Non-linear terms				
F-stat		2.91		12.17
P-value		0.06		0.00
N	140	140	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Test for Non-Linearity in the Price Phillips Curve

The regression table presents the results of testing for non-linearity in the price Phillips curve. The null hypothesis is that the non-linear terms are not significant. The findings indicate that the unemployment rate has a linear effect on price inflation. In contrast, the vacancy-to-unemployment ratio (V/U) exhibits borderline non-linearity in the pre-pandemic sample and is significantly non-linear in the full sample, with a p-value approaching zero.

	1985Q1-2019Q4			1985Q1-2023Q4		
	(1) U	(2) V/U	(3) Horse Race	(4) U	(5) V/U	(6) Horse Race
Unemp. Rate	0.43		-1.44	-3.62		9.15*
	(1.91)		(3.59)	(2.89)		(4.65)
U-squared	-0.12		0.10	0.48		-1.30**
	(0.31)		(0.51)	(0.44)		(0.66)
U-cubed	0.01		0.00	-0.02		0.06*
	(0.02)		(0.02)	(0.02)		(0.03)
V/U		4.73	13.54		2.59	-1.86
		(4.70)	(8.51)		(4.06)	(5.41)
(V/U)-squared		-4.61	-18.05		-1.88	4.61
		(6.96)	(12.17)		(5.26)	(7.27)
(V/U)-cubed		1.64	7.20		0.99	-1.09
		(3.17)	(5.09)		(1.79)	(2.40)
Constant	-0.18	-1.73*	1.42	8.96	-1.31	-21.25**
	(3.79)	(0.97)	(7.39)	(6.20)	(0.88)	(10.03)
R2	0.20	0.19	0.24	0.20	0.48	0.55
R2a	0.18	0.17	0.20	0.18	0.47	0.54
H0: U terms						
F-stat	7.47		1.31	3.55		1.32
P-value	0.00		0.27	0.02		0.27
H0: V/U terms						
F-stat		7.26	1.07		16.72	26.29
P-value		0.00	0.36		0.00	0.00
N	140	140	140	156	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Price Phillips Curve: 16% Trimmed Mean CPI

In this regression specification, the weighted median inflation rate is replaced with the Cleveland Fed's 16% trimmed mean inflation. The results are consistent with the baseline findings using the weighted median inflation. In the pre-pandemic sample, both measures are significant at nearly the 0% level and exhibit similar R-squared values. In the "horse race" comparison, the two measures do not outperform each other in terms of p-values. In the full sample, the model using the V/U ratio as the slack measure outperforms the model with the unemployment rate in terms of explanatory power (0.18 vs. 0.47). Additionally, V/U is significant with a p-value approaching zero, whereas the unemployment rate is not significant in column 6.

	1985Q1-2019Q4			1985Q1-2023Q4		
	(1) U	(2) V/U	(3) Horse Race	(4) U	(5) V/U	(6) Horse Race
Unemp. Rate	-2.04		-2.03	-6.49**		4.60
	(1.95)		(3.05)	(2.73)		(4.48)
U-squared	0.32		0.34	0.99**		-0.55
	(0.30)		(0.44)	(0.41)		(0.61)
U-cubed	-0.02		-0.02	-0.05**		0.02
	(0.01)		(0.02)	(0.02)		(0.03)
V/U		4.93	7.16		4.26	2.56
		(3.83)	(5.49)		(3.57)	(5.05)
(V/U)-squared		-4.37	-5.63		-3.88	0.55
		(5.94)	(7.63)		(4.84)	(7.18)
(V/U)-cubed		1.25	1.31		1.57	-0.06
		(2.81)	(3.22)		(1.70)	(2.45)
Constant	4.14	-1.78**	0.94	13.86**	-1.62**	-14.04
	(4.01)	(0.75)	(6.54)	(5.83)	(0.74)	(10.03)
R2	0.14	0.20	0.24	0.14	0.35	0.43
R2a	0.12	0.19	0.20	0.12	0.34	0.41
H0: U terms						
F-stat	5.87		1.79	6.20		1.39
P-value	0.00		0.15	0.00		0.25
H0: V/U terms						
F-stat		7.83	3.89		32.00	44.57
P-value		0.00	0.01		0.00	0.00
N	140	140	140	156	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Price Phillips Curve: CPI Excluding Food and Energy (CPIXFE)

Using CPIXFE inflation, the V/U ratio outperforms the unemployment rate in the pre-pandemic sample in the “horse race” comparison. It also performs better in the full sample in terms of statistical significance and explanatory power. However, the R-squared of the pre-pandemic model fitted with the unemployment rate is lower than that of the baseline model using the weighted median, and that of the previous model using the trimmed mean inflation rate.

	1985Q1-2019Q4			1985Q1-2023Q4		
	(1) U	(2) V/U	(3) Horse Race	(4) U	(5) V/U	(6) Horse Race
Unemp. Rate	-0.23		-0.46	-4.65		5.72
	(1.79)		(3.37)	(2.82)		(3.87)
U-squared	0.02		0.07	0.68		-0.77
	(0.28)		(0.47)	(0.43)		(0.54)
U-cubed	-0.00		-0.00	-0.03		0.04
	(0.01)		(0.02)	(0.02)		(0.02)
V/U		-2.00	1.51		1.01	2.66
		(3.52)	(8.28)		(3.24)	(5.31)
(V/U)-squared		4.45	0.18		-0.93	-0.11
		(5.92)	(11.92)		(4.44)	(6.90)
(V/U)-cubed		-2.39	-0.72		0.71	0.22
		(2.90)	(5.01)		(1.56)	(2.31)
Constant	0.12	-0.58	-0.55	9.73	-1.06*	-16.24*
	(3.60)	(0.59)	(7.04)	(5.99)	(0.63)	(8.58)
R2	0.01	0.03	0.04	0.07	0.28	0.38
R2a	-0.01	0.01	-0.01	0.05	0.26	0.35
H0: U terms						
F-stat	0.81		0.16	1.34		1.70
P-value	0.49		0.92	0.26		0.17
H0: V/U terms						
F-stat		1.64	0.56		41.88	51.35
P-value		0.18	0.64		0.00	0.00
N	140	140	140	156	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 11: Price Phillips Curve: PCE Excluding Food and Energy (PCEXFE)

Using PCEXFE inflation, the V/U ratio model ties with the unemployment rate model in the pre-pandemic sample. Similarly, in the full sample, the V/U ratio well outperforms the unemployment rate in both statistical significance and explanatory power.

B Appendix: Regional Evidence

B.1 List of MSAs

Atlanta-Sandy Springs-Roswell, GA; Chicago-Naperville-Elgin, IL-IN-WI; Dallas-Fort Worth-Arlington, TX; Denver-Aurora-Lakewood, CO; Detroit-Warren-Dearborn, MI; Houston-The Woodlands-Sugar Land, TX; Los Angeles-Long Beach-Anaheim, CA; Miami-Fort Lauderdale-West Palm Beach, FL; Minneapolis-St. Paul-Bloomington, MN-WI; New York-Newark-Jersey City, NY-NJ-PA; Philadelphia-Camden-Wilmington, PA-NJ-DE-MD; Phoenix-Mesa-Scottsdale, AZ; Riverside-San Bernardino-Ontario, CA; San Diego-Carlsbad, CA; San Francisco-Oakland-Hayward, CA; Seattle-Tacoma-Bellevue, WA; Washington-Arlington-Alexandria, DC-VA-MD-WV; and Boston-Cambridge-Nashua, MA-NH NECTA.

B.2 Extended Kalman Filter Results

Estimated V/U Star

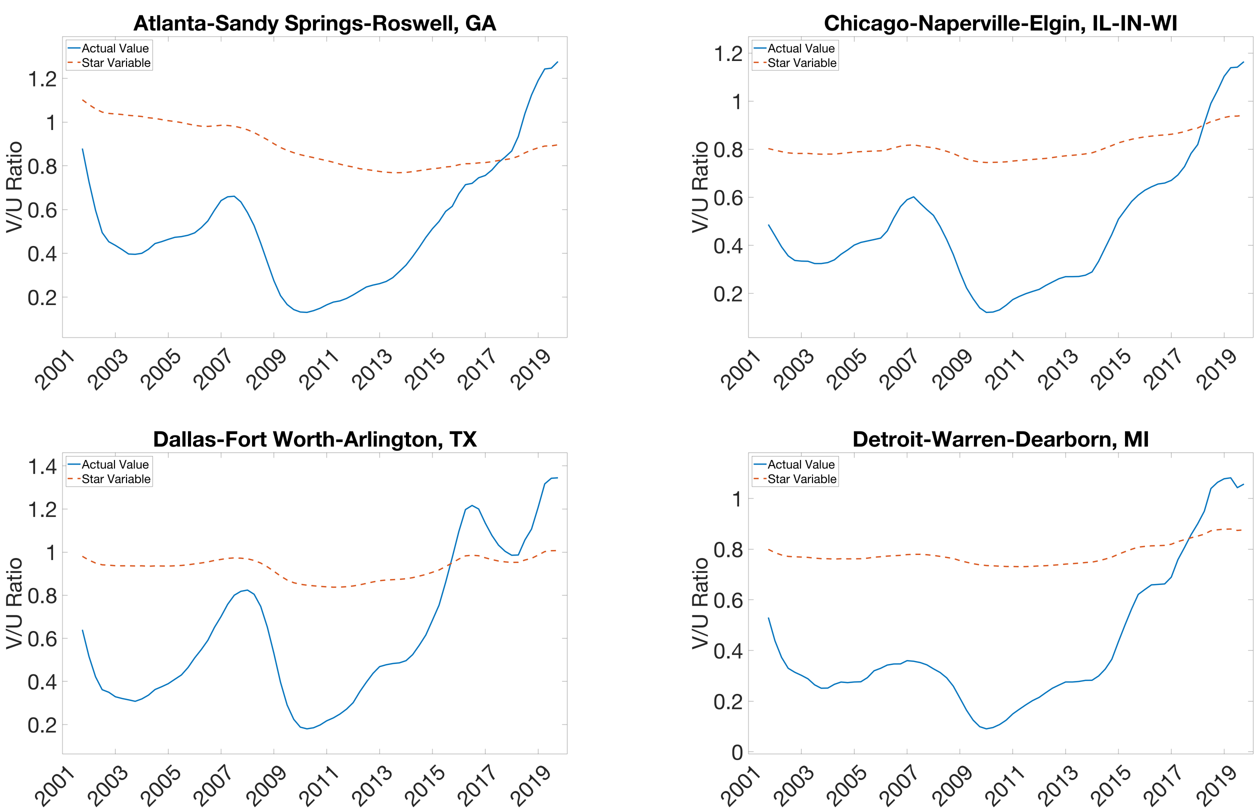


Figure 10: Natural Rate vs. Actual V/U Values for MSAs: Atlanta, Chicago, Dallas, Detroit

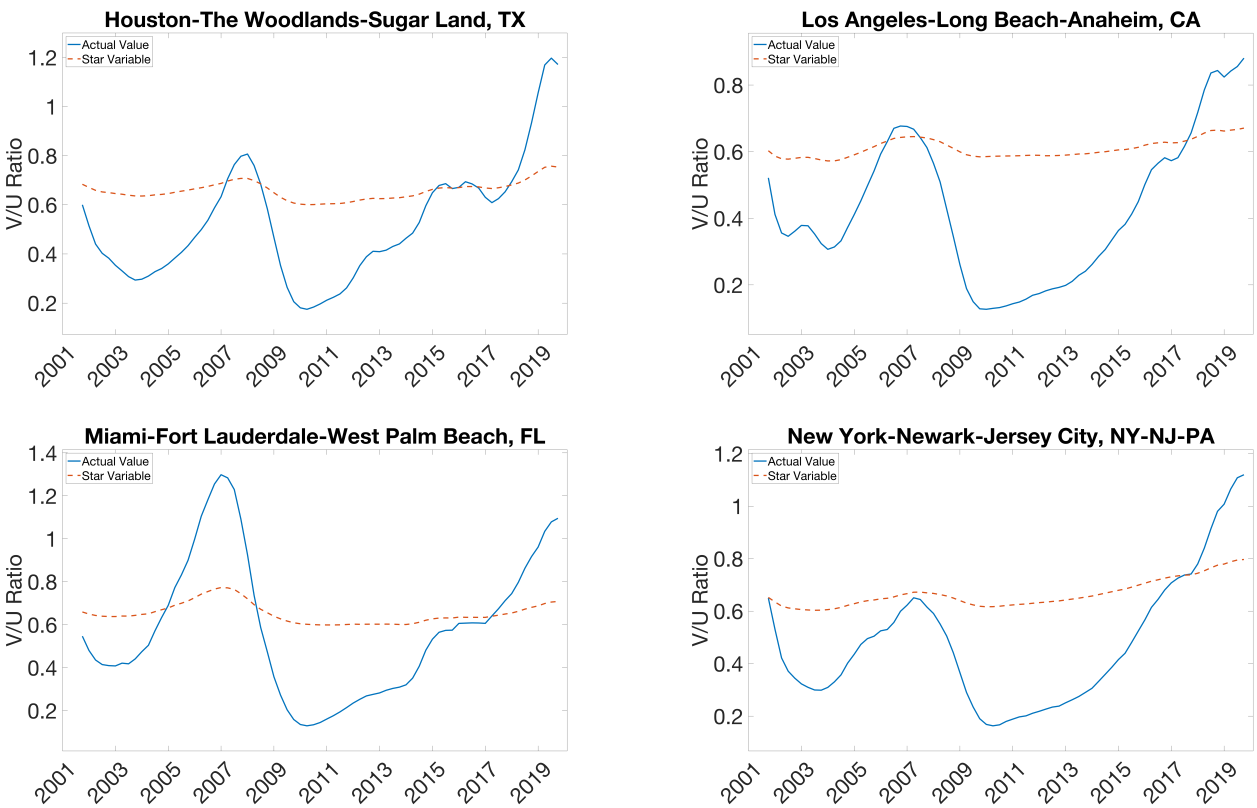


Figure 11: Natural Rate vs. Actual V/U Values for MSAs: Houston, Los Angeles, Miami, New York

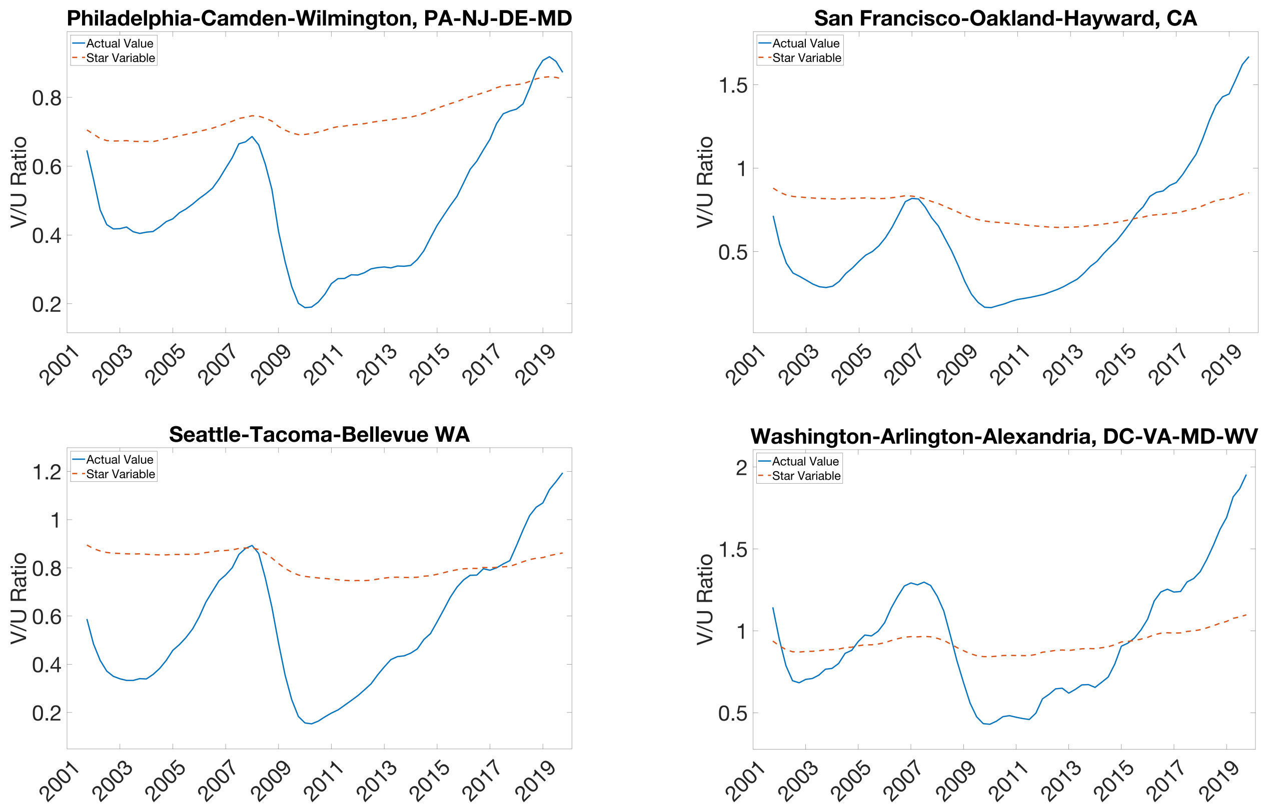


Figure 12: Natural Rate vs. Actual V/U Values for MSAs: Philadelphia, San Francisco, Seattle, Washington DC

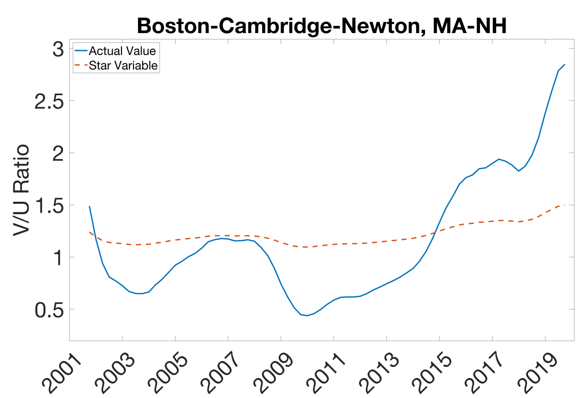


Figure 13: Natural Rate vs. Actual V/U Values for MSA: Boston

Fitted Inflation Rate with V/U Gap

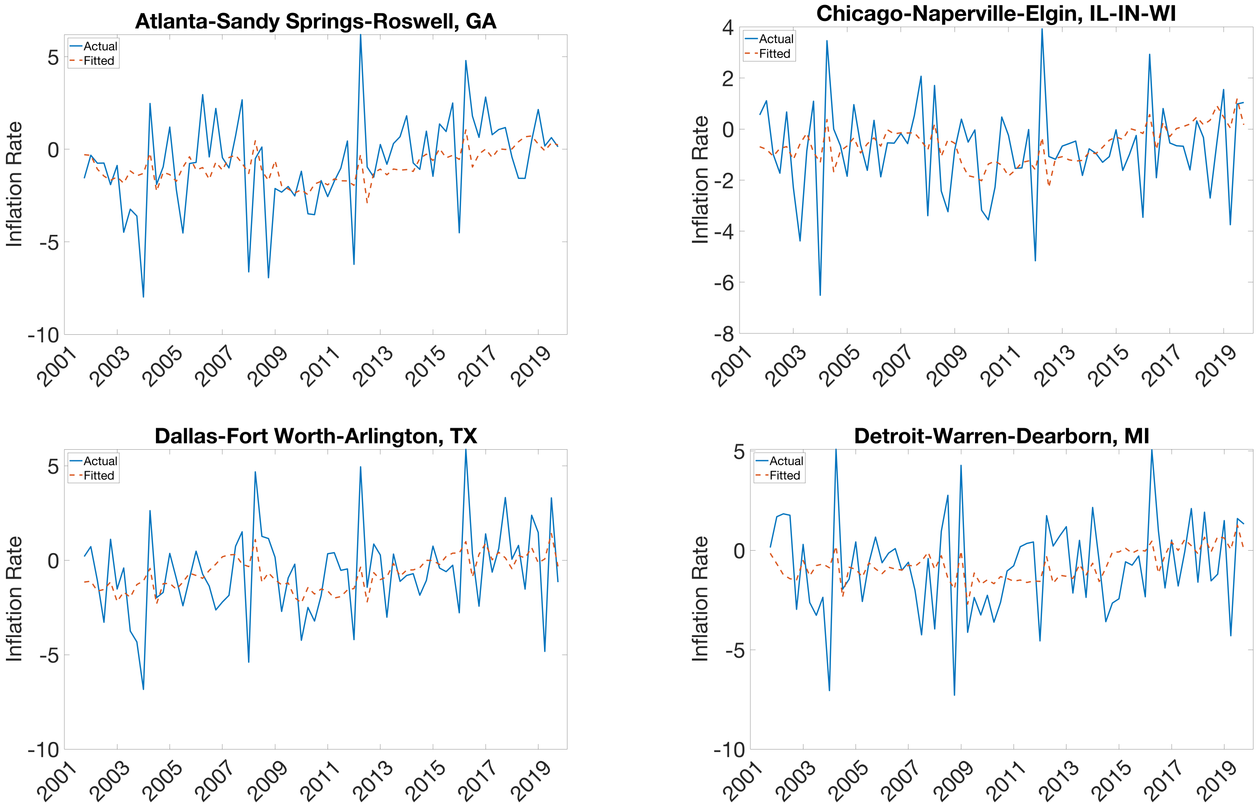


Figure 14: Fitted Inflation Rate (V/U) for MSAs: Atlanta, Chicago, Dallas, Detroit

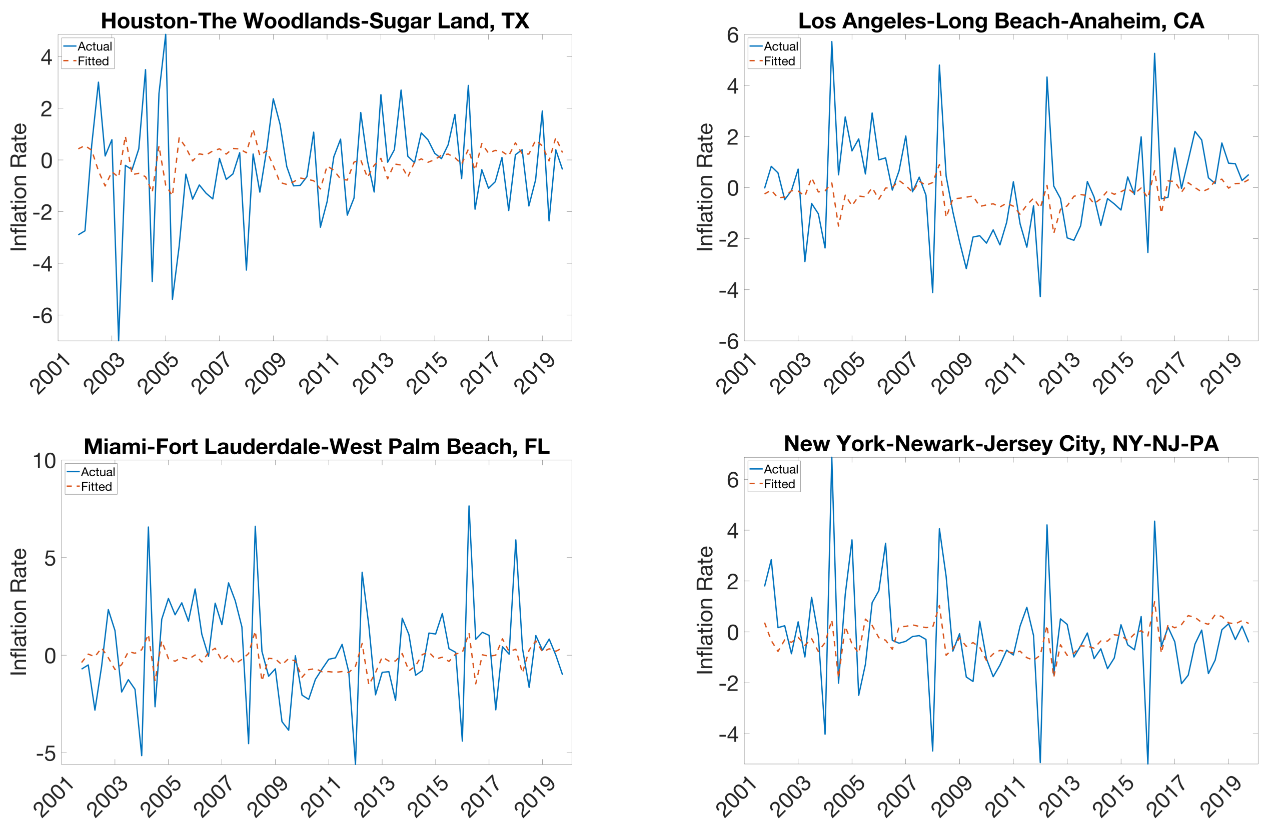


Figure 15: Fitted Inflation Rate (V/U) for MSAs: Houston, Los Angeles, Miami, New York

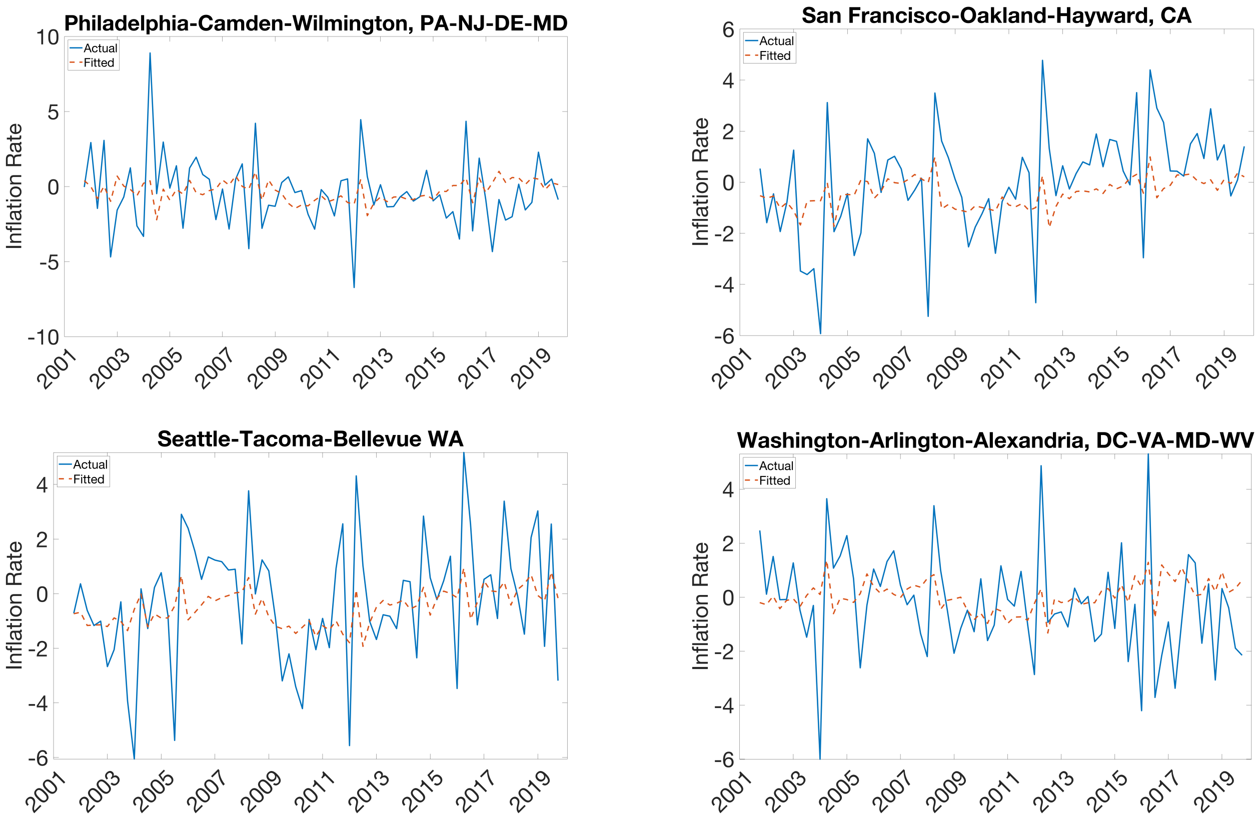


Figure 16: Fitted Inflation Rate (V/U) for MSAs: Philadelphia, San Francisco, Seattle, Washington DC

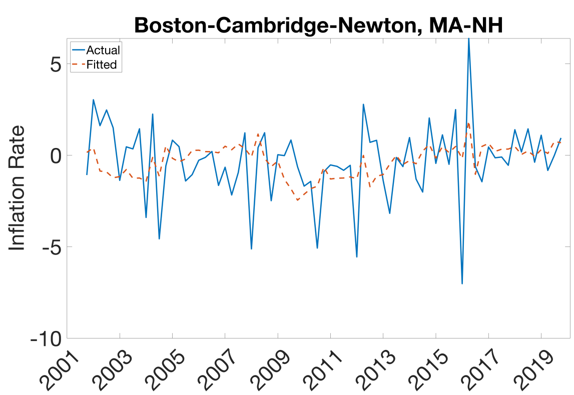


Figure 17: Fitted Inflation Rate (V/U) for MSA: Boston

Estimated U Star

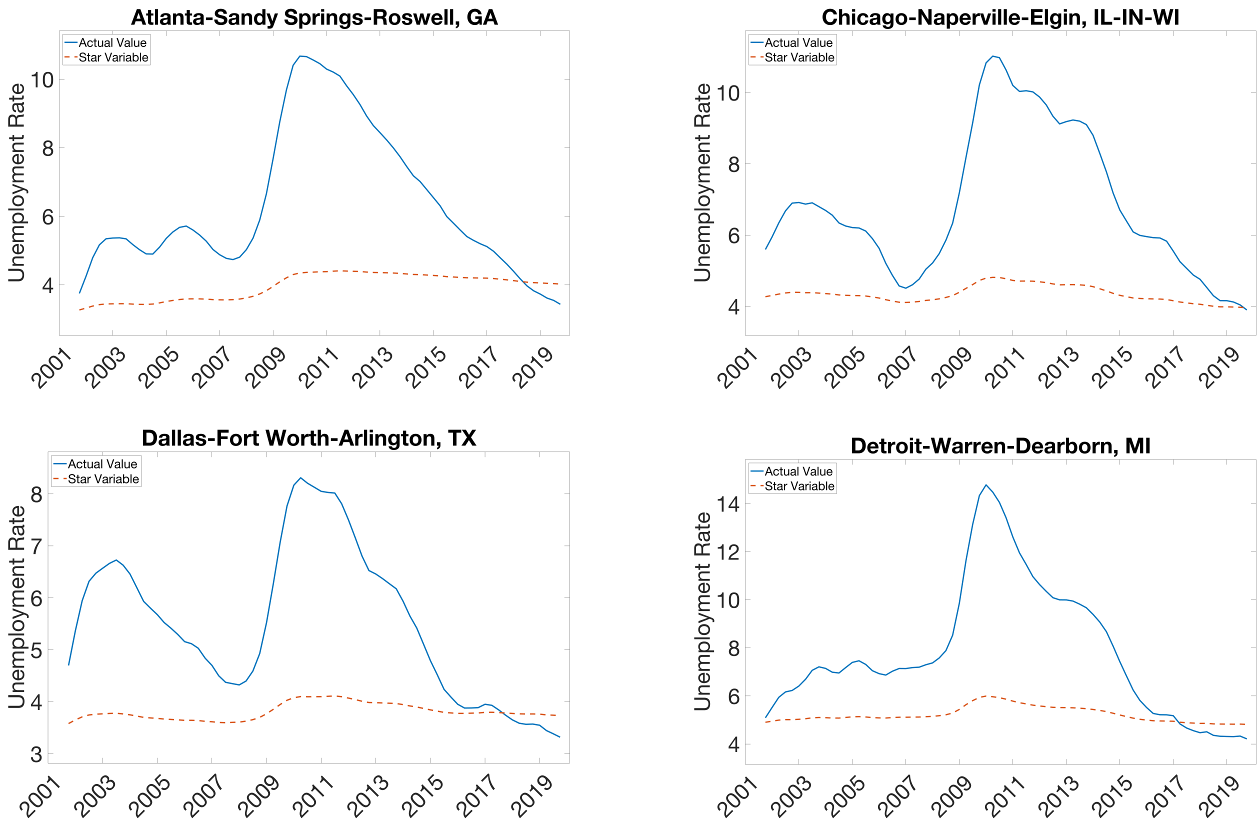


Figure 18: Natural Rate vs. Actual U Values for MSAs: Atlanta, Chicago, Dallas, Detroit

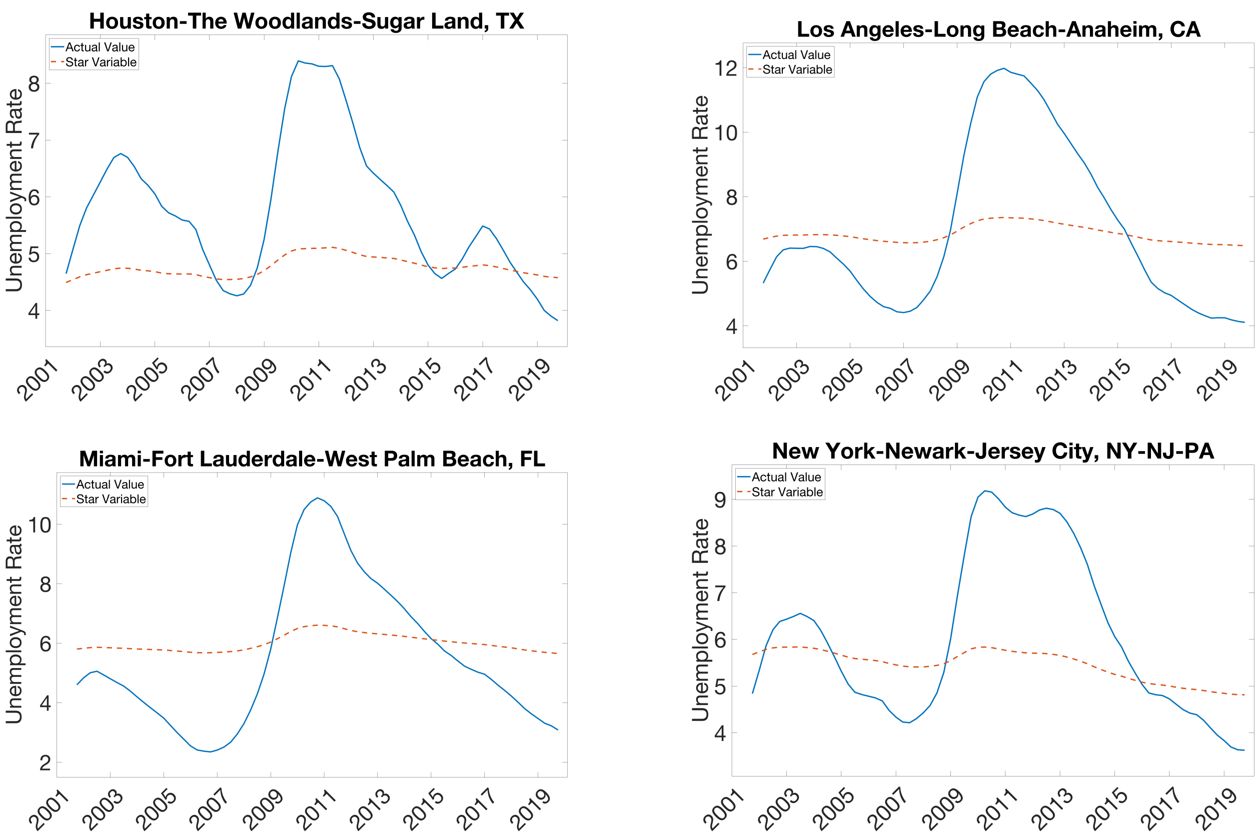


Figure 19: Natural Rate vs. Actual U Values for MSAs: Houston, Los Angeles, Miami, New York

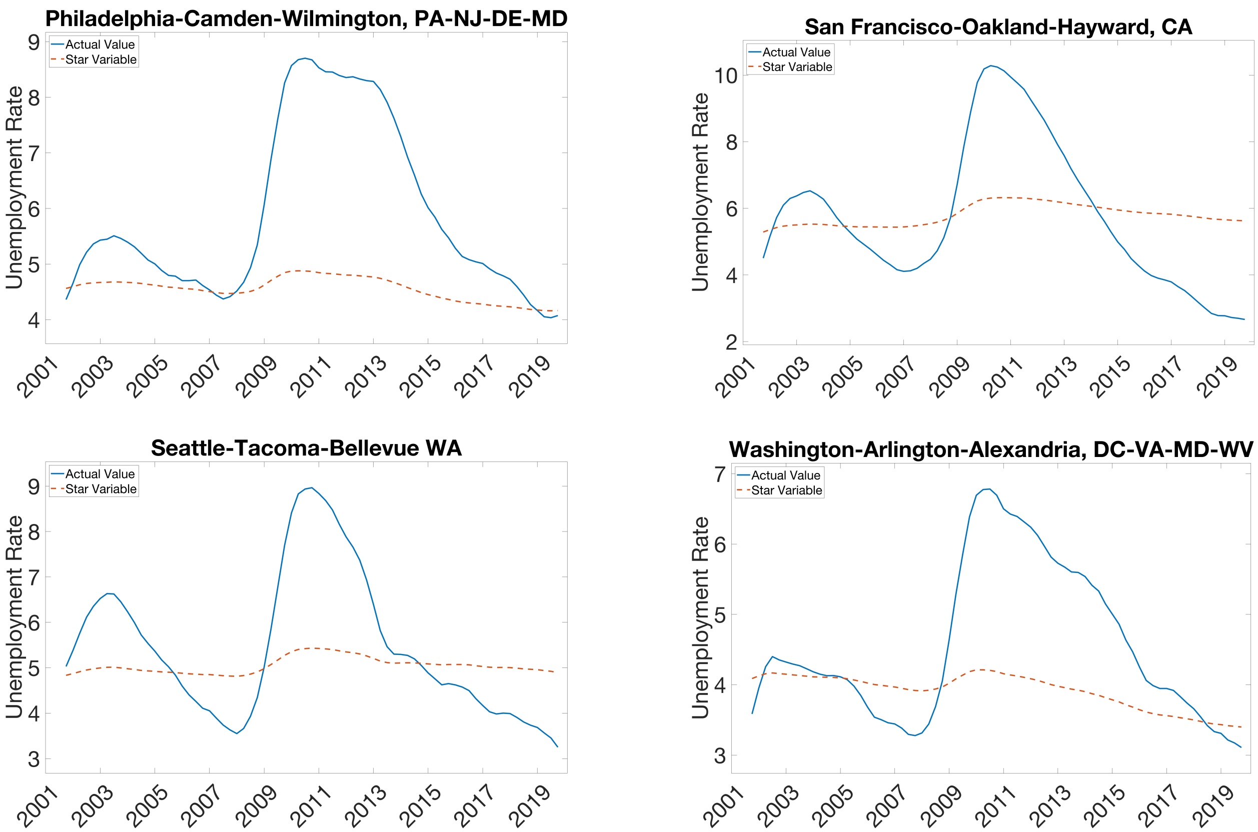


Figure 20: Natural Rate vs. Actual U Values for MSAs: Philadelphia, San Francisco, Seattle, Washington DC

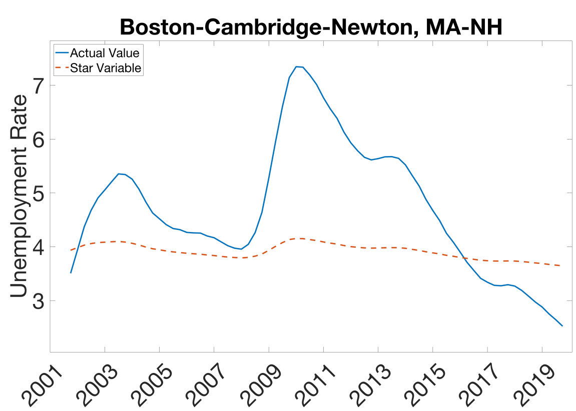


Figure 21: Natural Rate vs. Actual U Values for MSA: Boston

Fitted Inflation Rate with U Gap

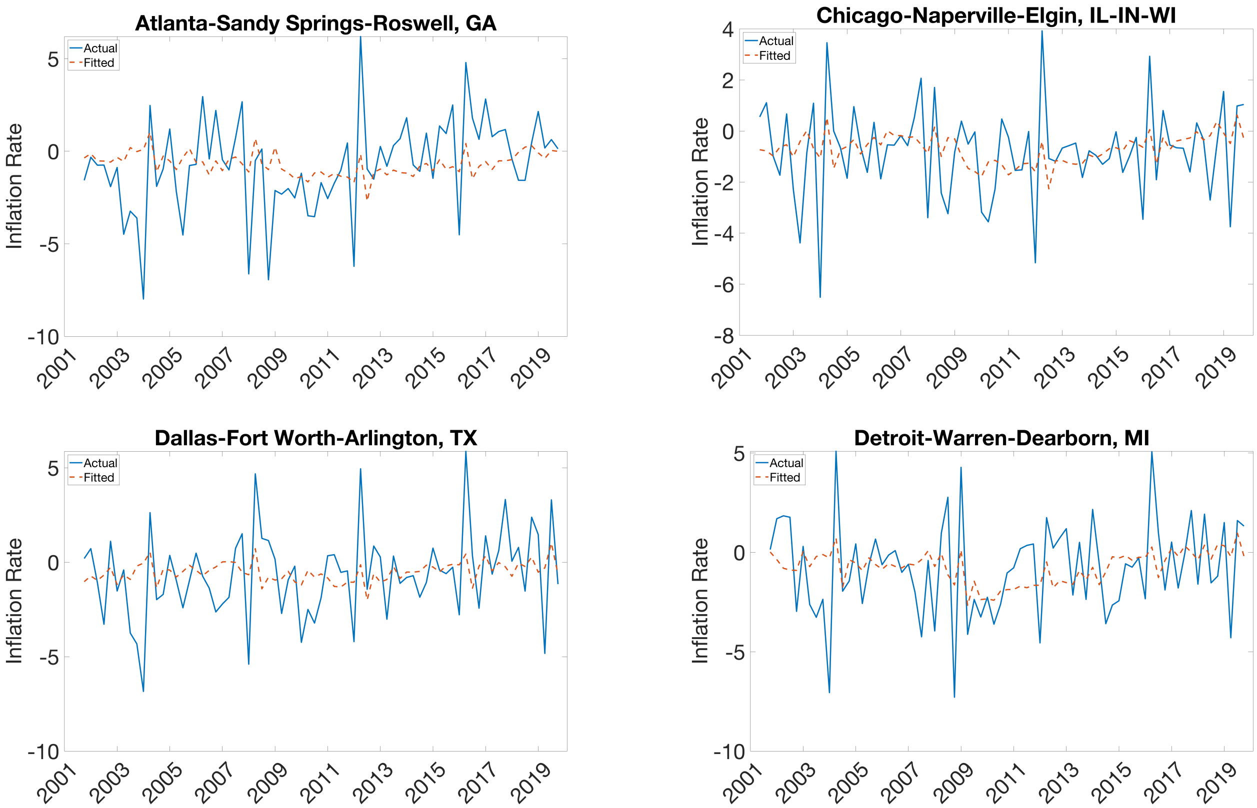


Figure 22: Fitted Inflation Rate (U) for MSAs: Atlanta, Chicago, Dallas, Detroit

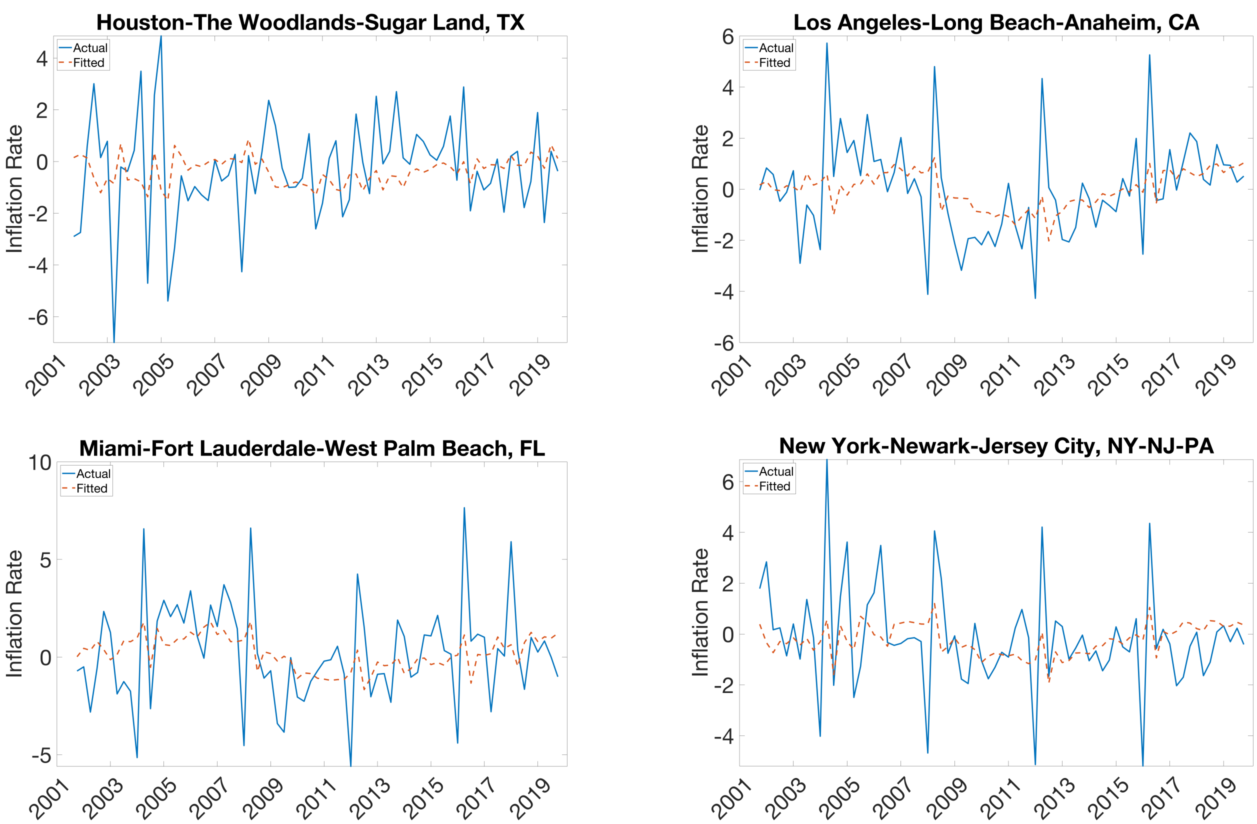


Figure 23: Fitted Inflation Rate (U) for MSAs: Houston, Los Angeles, Miami, New York

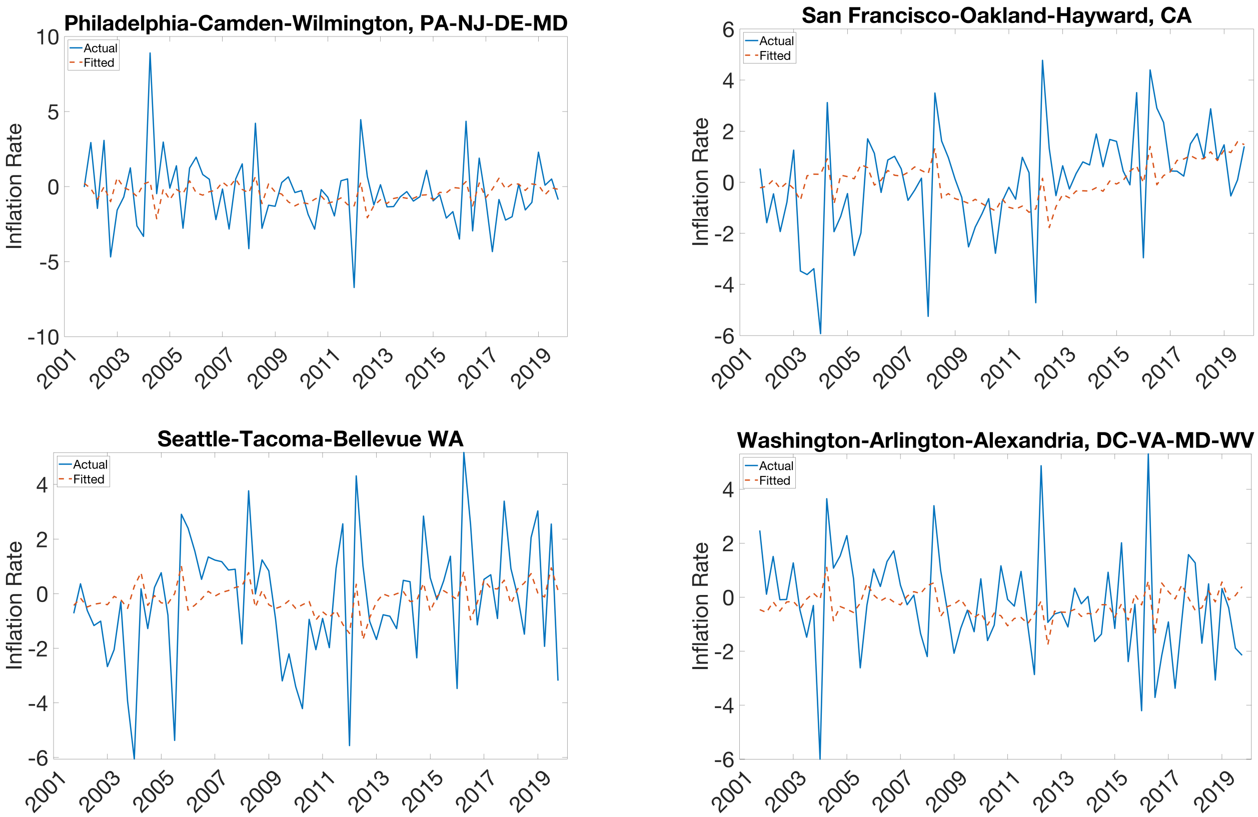


Figure 24: Fitted Inflation Rate (U) for MSAs: Philadelphia, San Francisco, Seattle, Washington DC

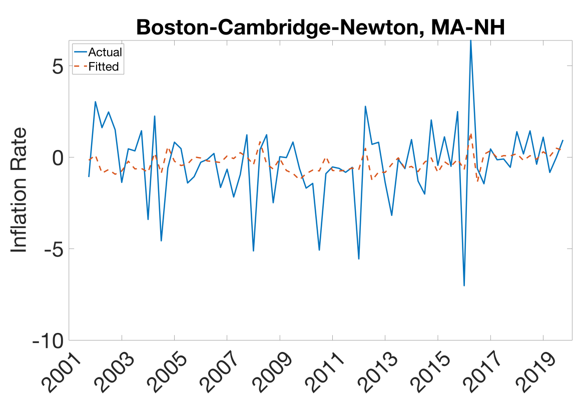


Figure 25: Fitted Inflation Rate (U) for MSA: Boston

C Appendix: Wage Analysis

C.1 Test for Non-linearity

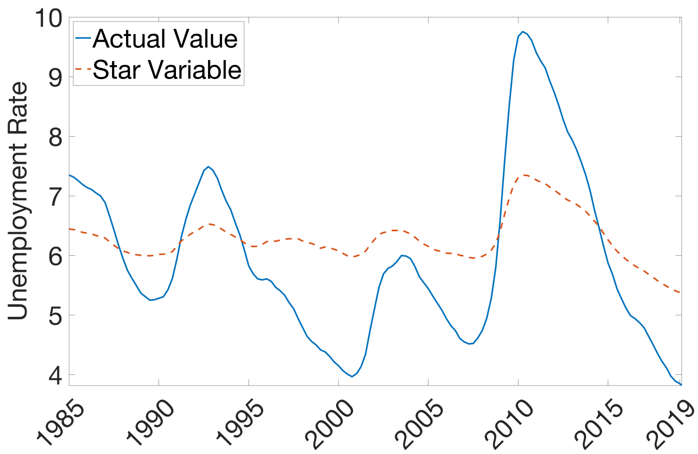
	1985Q1-2019Q4		1985Q1-2023Q4	
	(1) U	(2) V/U	(3) U	(4) V/U
Unemp. Rate	-0.78		-2.80	
	(1.41)		(1.80)	
U-squared	-0.01		0.29	
	(0.22)		(0.29)	
U-cubed	0.00		-0.01	
	(0.01)		(0.01)	
V/U		-1.15		1.92
		(2.22)		(1.78)
(V/U)-squared		6.02*		0.55
		(3.49)		(2.28)
(V/U)-cubed		-3.33**		-0.30
		(1.65)		(0.79)
Labor Prod.	0.30***	0.59***	0.15	0.54***
	(0.08)	(0.09)	(0.15)	(0.11)
Constant	3.44	-1.80***	8.21**	-2.18***
	(2.86)	(0.33)	(3.51)	(0.37)
R2	0.54	0.51	0.45	0.55
R2a	0.52	0.50	0.43	0.54
H0: U Non-linear terms				
F-stat	11.38		10.39	
P-value	0.00		0.00	
H0: V/U Non-linear terms				
F-stat		3.05		0.76
P-value		0.05		0.47
N	140	140	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

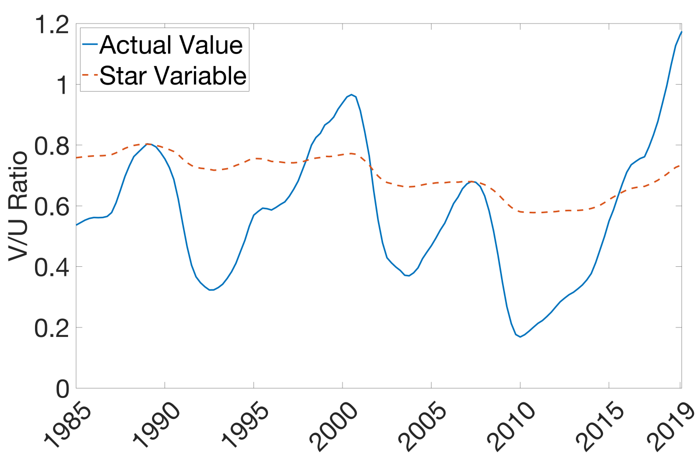
Table 12: Test for Non-Linearity in the Wage Phillips Curve

The regression table presents the results of testing for non-linearity in the wage Phillips curve. The findings indicate that the effect of the unemployment rate on wage inflation is non-linear across both sample periods. The vacancy-to-unemployment ratio (V/U) has a non-linear relationship with wage inflation at the 5% significance level in the pre-pandemic sample, but is largely linear in the full sample.

C.2 Wage UKF



(a) U: Natural Rate vs. Actual Value



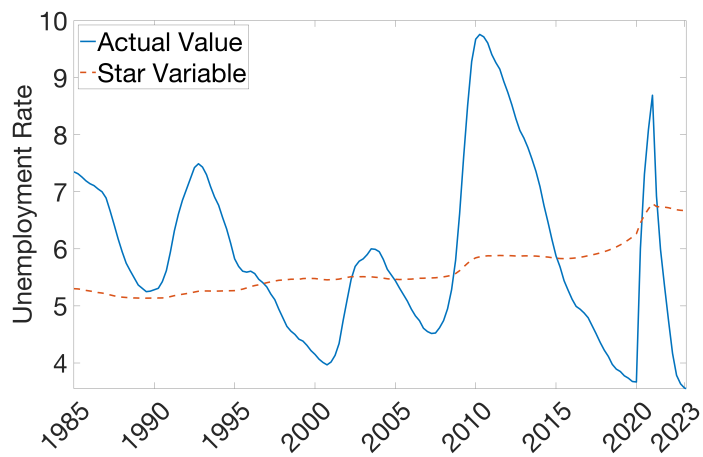
(b) V/U: Natural Rate vs. Actual Value

Figure 26: Wage Inflation: UKF-Estimated Star Variables, 1985–2019Q4

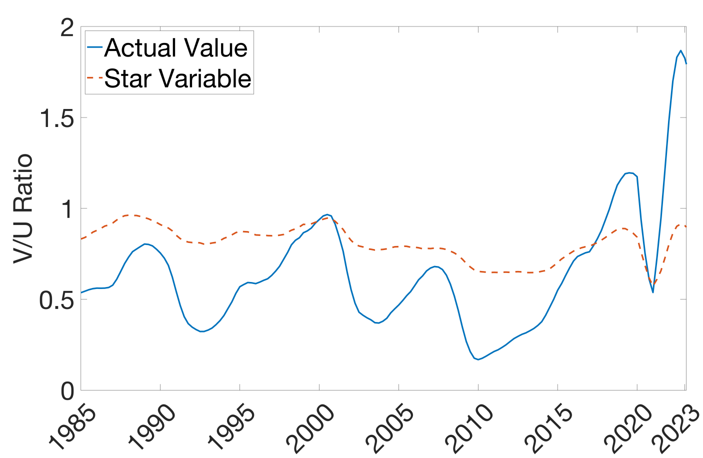
	1985Q1-2019Q4		1985Q1-2023Q4	
	(1) Unemp. Rate	(2) V/U Ratio	(3) Unemp. Rate	(4) V/U Ratio
motion eq.				
$(S - S^{*})_{t-1}$	0.9981***	0.9986***	0.9953***	0.9909***
	(0.02)	(0.02)	(0.02)	(0.02)
measurement eq.				
slack gap	-0.6244***	2.1806***	-0.3962***	2.7381***
	(0.11)	(0.45)	(0.08)	(0.48)
slack gap^2	0.1409***	-2.6521***	0.0441*	-0.7785
	(0.05)	(0.96)	(0.03)	(0.71)
slack gap^3	0.0310	-0.7955	-0.0043	-0.3651
	(0.03)	(1.56)	(0.01)	(1.33)
labor productivity	0.1211	0.5856***	0.2654*	0.6837***
	(0.12)	(0.09)	(0.14)	(0.11)
constant	-0.5679**	-0.6660***	-0.2887	-0.5595**
	(0.23)	(0.22)	(0.30)	(0.23)
error variances				
S^{*} eq.	0.01390	0.00021	0.00750	0.00110
	(restricted)	(restricted)	(restricted)	(restricted)
$S - S^{*}$ eq.	0.0369***	0.0011***	0.1253***	0.0028***
	(0.00)	(0.00)	(0.01)	(0.00)
PC eq.	0.2995***	0.2908***	0.5002***	0.3858***
	(0.04)	(0.03)	(0.04)	(0.04)
$H_0 : \beta_2 = \beta_3 = 0$				
Wald-stat	10.01	17.38	3.28	3.86
P-value	(0.01)	(0.00)	(0.19)	(0.14)
$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$				
Wald-stat	81.20	89.27	85.45	113.44
P-value	(0.00)	(0.00)	(0.00)	(0.00)
R2a	0.55	0.54	0.50	0.63
N	140	140	156	156

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13: Wage Phillips Curve Estimation Results with Time-varying Natural Rates



(a) U: Natural Rate vs. Actual Value



(b) V/U: Natural Rate vs. Actual Value

Figure 27: Wage Inflation: UKF-Estimated Star Variables, 1985–2023Q4

D Appendix: Model

D.1 Wage Function

Want to show that w is an increasing function of θ.

Proof. Recall that

$$w = \frac{\beta\left(r+s+a\theta^{1-\gamma}\right)}{(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\beta\cdot\left(r+s+a\theta^{1-\gamma}\right)}\cdot y,$$

taking first derivative,

$$\begin{aligned}\frac{d(\text{numerator})}{d\theta} &= \beta(1-\gamma)a\theta^{-\gamma}\\ \frac{d(\text{denominator})}{d\theta} &= -(1-\beta)\gamma a\theta^{-\gamma-1}+\beta(1-\gamma)a\theta^{-\gamma}\end{aligned}$$

Leaving y out, we have the numerator of $dw/d\theta$ as

$$\begin{aligned}&\beta(1-\gamma)a\theta^{-\gamma}\cdot\left[(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\beta\left(r+s+a\theta^{1-\gamma}\right)\right]\\ &\quad-\beta\left(r+s+a\theta^{1-\gamma}\right)\left[-(1-\beta)\gamma a\theta^{-\gamma-1}+\beta(1-\gamma)a\theta^{-\gamma}\right]\\ =&\beta(1-\gamma)a\theta^{-\gamma}(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\underbrace{\beta^2(1-\gamma)a\theta^{-\gamma}\left(r+s+a\theta^{1-\gamma}\right)}_{\text{cancels out}}\\ &\quad+\beta\left(r+s+a\theta^{1-\gamma}\right)(1-\beta)\gamma a\theta^{-\gamma-1}\underbrace{-\beta^2\left(r+s+a\theta^{1-\gamma}\right)(1-\gamma)a\theta^{-\gamma}}_{\text{cancels out}}\\ =&\beta(1-\gamma)a\theta^{-\gamma}(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\beta\left(r+s+a\theta^{1-\gamma}\right)(1-\beta)\gamma a\theta^{-\gamma-1}.\end{aligned}$$

Thus, the first derivative is

$$dw/d\theta = \frac{\beta(1-\gamma)a\theta^{-\gamma}(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\beta\left(r+s+a\theta^{1-\gamma}\right)(1-\beta)\gamma a\theta^{-\gamma-1}}{\left[(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\beta\cdot\left(r+s+a\theta^{1-\gamma}\right)\right]^2}.$$

Since the denominator will always be positive, $dw/d\theta > 0$ given

- β is positive between 0 and 1; $\theta > 0$
- $(1-\gamma) > 0$ with CRS assumption.

□

D.2 Log-linearization

D.2.1 Job Creation Condition

$$
\begin{aligned}
v &= J/L - 1 + u \\
log v &= log\left(J/L - 1 + u\right) \\
\frac{dv}{v} &= \frac{du}{J/L - 1 + u} \\
\widetilde{v} &= \frac{du}{\bar{v}} \\
\widetilde{v} &= \frac{\bar{u}}{\bar{v}} \cdot \frac{du}{\bar{u}} \\
\widetilde{v} &= \frac{\bar{u}}{\bar{v}} \cdot \widetilde{u} \\
\widetilde{u} &= \bar{\theta} \cdot \widetilde{v}
\end{aligned}
$$

D.2.2 Labor Market Tightness

$$
\begin{aligned}
\theta &= v/u \\
log\theta &= log v - log u \\
d\theta/\bar{\theta} &= \widetilde{v} - \widetilde{u} \\
\widetilde{\theta} &= \widetilde{v} - \widetilde{u}
\end{aligned}
$$

D.2.3 Beveridge Curve

$$
\begin{aligned}
v &= a^{-\frac{1}{1-\gamma}} \cdot [s(1-u)]^{\frac{1}{1-\gamma}} \cdot u^{-\frac{\gamma}{1-\gamma}} \\
\log v &= \log\left(a^{-\frac{1}{1-\gamma}} [s(1-u)]^{\frac{1}{1-\gamma}} u^{-\frac{\gamma}{1-\gamma}}\right) \\
\frac{dv}{v} &= -\frac{1}{1-\gamma}\frac{da}{a} + \frac{1}{1-\gamma}\frac{ds}{s} + \frac{1}{1-\gamma}\left(\frac{-du}{1-u}\right) - \frac{\gamma}{1-\gamma}\frac{du}{u} \\
\widetilde{v} &= -\frac{1}{1-\gamma}\widetilde{a} + \frac{1}{1-\gamma}\widetilde{s} - \frac{u \cdot du}{(1-\gamma)(1-u)u} - \frac{\gamma}{1-\gamma}\widetilde{u} \\
\widetilde{v} &= -\frac{1}{1-\gamma}\widetilde{a} + \frac{1}{1-\gamma}\widetilde{s} - \frac{1}{1-\gamma}\frac{u}{1-u}\widetilde{u} - \frac{\gamma}{1-\gamma}\widetilde{u} \\
\widetilde{v} &= -\frac{1}{1-\gamma}\widetilde{a} - \frac{1}{1-\gamma}\left(\frac{\bar{u}}{1-\bar{u}} - \gamma\right)\widetilde{u} \text{ , since shock to matching efficiency.}
\end{aligned}
$$

D.2.4 Wage Function

$$w = \frac{\beta\left(r+s+a\theta^{1-\gamma}\right)}{(1-\beta)\left(r+s+a\theta^{-\gamma}\right)+\beta\left(r+s+a\theta^{1-\gamma}\right)} \text{, where } y \text{ is normalized to 1}$$

$$[\widetilde{w}] + [r+s+a(1\widetilde{-\beta)\theta}^{-\gamma}+\beta a\theta^{1-\gamma}] = [\beta\,(r+\widetilde{s+a}\theta^{1-\gamma})]$$

It follows that percentage deviation of $a(1-\beta)\theta^{-\gamma}+a\beta\theta^{1-\gamma}$ can be derived as:

$$\begin{aligned}
&= \frac{\left[a(1-\beta)\bar{\theta}^{-}+a\beta\theta^{1-\gamma}\right]-\left[\bar{a}(1-\beta)\bar{\theta}^{-\gamma}+a\beta(\bar{\theta})^{1-\gamma}\right]}{\bar{a}(1-\beta)\bar{\theta}^{-\gamma}+\bar{a}\beta\bar{\theta}^{1-\gamma}} \\
&= \frac{(1-\beta)\left[a\theta^{-\gamma}-\bar{a}\bar{\theta}^{-\gamma}\right]+\beta\left(a\theta^{1-\gamma}-\bar{a}\bar{\theta}^{1-\gamma}\right)}{(1-\beta)\bar{a}\bar{\theta}^{-\gamma}+\beta\bar{a}\bar{\theta}^{1-\gamma}}
\end{aligned}$$

Following multivariate first order taylor expansion,

$$\begin{aligned}
a\theta^{-\gamma} &\approx \bar{a}\bar{\theta}^{-\gamma}+\left(\bar{\theta}\right)^{-\gamma}(a-\bar{a})+(-\gamma)\bar{a}(\bar{\theta})^{-\gamma-1}(\theta-\bar{\theta}) \\
a\theta^{-\gamma}-\bar{a}(\bar{\theta})^{-\gamma} &\approx (\bar{\theta})^{-\gamma}(a-\bar{a})-\gamma\bar{a}\bar{\theta}^{-\gamma}(\theta-\bar{\theta})\cdot\left(1/\bar{\theta}\right) \\
\left(1/\bar{a}\bar{\theta}\right)\left(a\theta^{-\gamma}-\bar{a}(\bar{\theta})^{-\gamma}\right) &\approx (\bar{\theta})^{-\gamma-1}\cdot\frac{a-\bar{a}}{\bar{a}}-\gamma(\bar{\theta})^{-\gamma-1}\cdot\frac{\theta-\bar{\theta}}{\bar{\theta}},
\end{aligned}$$

and thus,

$$\begin{aligned}
a\theta^{-\gamma}-\bar{a}(\bar{\theta})^{-\gamma} &\approx \left[(\bar{\theta})^{-\gamma-1}\tilde{a}-\gamma(\bar{\theta})^{-\gamma-1}\tilde{\theta}\right]\bar{a}\bar{\theta} \\
&\approx \bar{a}(\bar{\theta})^{-\gamma}\tilde{a}-\gamma\bar{a}(\bar{\theta})^{-\gamma}\tilde{\theta}.
\end{aligned}$$

Going back to the percentage deviation of $a(1-\beta)\theta^{-\gamma}+a\beta\theta^{1-\gamma}$, we have:

$$= \frac{(1-\beta)\left[\bar{a}(\bar{\theta})^{-\gamma}\tilde{a}-\gamma\bar{a}(\bar{\theta})^{-\gamma}\tilde{\theta}\right]+\beta\left[\bar{a}(\bar{\theta})^{1-\gamma}\tilde{a}+(1-\gamma)\bar{a}(\bar{\theta})^{1-\gamma}\tilde{\theta}\right]}{(1-\beta)\bar{a}\bar{\theta}^{-\gamma}+\beta\bar{a}\bar{\theta}^{1-\gamma}}$$

Thus,

$$\begin{aligned}
&[r+s+a(1\widetilde{-\beta)\theta}^{-\gamma}+\beta a\theta^{1-\gamma}] \\
&= \frac{\left[r+s+a(1-\beta)\theta^{-\gamma}+a\beta\theta^{1-\gamma}\right]-\left[r+s+\bar{a}(1-\beta)\bar{\theta}^{-\gamma}+\beta\bar{a}\bar{\theta}^{1-\gamma}\right]}{r+s+\bar{a}(1-\beta)\bar{\theta}^{-\gamma}+\beta\bar{a}\bar{\theta}^{1-\gamma}} \\
&= \frac{\left[(1-\beta)\bar{a}\bar{\theta}^{-\gamma}+\beta\bar{a}\bar{\theta}^{1-\gamma}\right]}{r+s+\bar{a}(1-\beta)\bar{\theta}^{-\gamma}+\bar{a}\beta\bar{\theta}^{1-\gamma}}\cdot\left[\frac{(1-\beta)\left[\bar{a}(\bar{\theta})^{-\gamma}\tilde{a}-\gamma\bar{a}(\bar{\theta})^{-\gamma}\tilde{\theta}\right]+\beta\left[\bar{a}(\bar{\theta})^{1-\gamma}\tilde{a}+(1-\gamma)\bar{a}(\bar{\theta})^{1-\gamma}\tilde{\theta}\right]}{(1-\beta)\bar{a}\bar{\theta}^{-\gamma}+\beta\bar{a}\bar{\theta}^{1-\gamma}}\right]
\end{aligned}$$

For the right-hand side,

$$[\beta\,(r+\widetilde{s+a}\theta^{1-\gamma})] = \frac{\bar{a}\bar{\theta}^{1-\gamma}\tilde{a} + (1-\gamma)\bar{a}\bar{\theta}^{-\gamma}\tilde{\theta}}{r+s+\bar{a}\bar{\theta}^{1-\gamma}}$$

Therefore, the log-linearized wage function is:

$$\begin{aligned}
\widetilde{w} &= \frac{\bar{a}\bar{\theta}^{1-\gamma}\tilde{a} + (1-\gamma)\bar{a}\bar{\theta}^{-\gamma}\tilde{\theta}}{r+s+\bar{a}\bar{\theta}^{1-\gamma}} \cdot \beta \\
&\quad - \frac{\left[(1-\beta)\bar{a}\bar{\theta}^{-\gamma} + \beta\bar{a}\bar{\theta}^{1-\gamma}\right]}{r+s+\bar{a}(1-\beta)\bar{\theta}^{-\gamma} + \bar{a}\beta\bar{\theta}^{1-\gamma}} \cdot \left[\frac{(1-\beta)\left[\bar{a}(\bar{\theta})^{-\gamma}\tilde{a} - \gamma\bar{a}(\bar{\theta})^{-\gamma}\tilde{\theta}\right] + \beta\left[\bar{a}(\bar{\theta})^{1-\gamma}\tilde{a} + (1-\gamma)\bar{a}(\bar{\theta})^{1-\gamma}\tilde{\theta}\right]}{(1-\beta)\bar{a}\bar{\theta}^{-\gamma} + \beta\bar{a}\bar{\theta}^{1-\gamma}}\right] \\
&= \frac{\bar{a}\bar{\theta}^{1-\gamma}\tilde{a} + (1-\gamma)\bar{a}\bar{\theta}^{-\gamma}\tilde{\theta}}{r+s+\bar{a}\bar{\theta}^{1-\gamma}} - \frac{(1-\beta)\left[\bar{a}(\bar{\theta})^{-\gamma}\tilde{a} - \gamma\bar{a}(\bar{\theta})^{-\gamma}\cdot\tilde{\theta}\right] + \beta\left[\bar{a}(\bar{\theta})^{1-\gamma}\tilde{a} + (1-\gamma)\bar{a}(\bar{\theta})^{1-\gamma}\cdot\tilde{\theta}\right]}{r+s+\bar{a}(1-\beta)\bar{\theta}^{-\gamma} + \bar{a}\beta\bar{\theta}^{1-\gamma}}
\end{aligned}$$